AD-A112 040 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 20/4
COMPUTER PROGRAM FOR CALCULATING HYDRODYNAMIC PROPERTIES OF SHO--ETC(U)
FEB 82 A E FUHS
UNCLASSIFIED NPS67-82-001 NL

1 OF 2
AD A
112 040

END
DATE
FILMED
04-82
DTIC

CONT.

1 OF 2

AD A

112 040

1.0 2.8 2.5

3.2 2.2

2.0

1.1

1.8

1.25 1.4 1.6

MICROCOPY RESOLUTION TEST CHART

(2)

NPS67-82-001

NAVAL POSTGRADUATE SCHOOL

Monterey, California

COMPUTER PROGRAM FOR CALCULATING
HYDRODYNAMIC PROPERTIES OF
SHOCK WAVES IN SEA WATER

Allen E. Fuhs

February 1982

Approved for public release; distribution unlimited.

Prepared for:

Mr. Donald Phillips
Naval Surface Weapons Center
White Oak Laboratories
Silver Spring, MD 20910

82

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral J. J. Ekelund
Superintendent

D. A. Shrady
Acting Provost

The work presented in this report is in support of warhead research sponsored by the Naval Surface Weapons Center. The results will be used in the development of an extensive computer code for calculating shaped charge penetration in sea water.

Reproduction of all or part of this report is authorized.

This report was prepared by:

Allen E. Fuhs

ALLEN E. FUHS
Distinguished Professor

Reviewed by:

Max F. Platzer

M. F. PLATZER
Chairman of Aeronautics

Released by:

William M. Tolles

W. M. TOLLES
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS67-82-001	2. GOVT ACCESSION NO. AD-A112	3. RECIPIENT'S CATALOG NUMBER 040
4. TITLE (and Subtitle) COMPUTER PROGRAM FOR CALCULATING HYDRODYNAMIC PROPERTIES OF SHOCK WAVES IN SEA WATER		5. TYPE OF REPORT & PERIOD COVERED Progress Report
		6. PERFORMING ORG. REPORT NUMBER NPS67-82-001
7. AUTHOR(s) Allen E. Fuhs		8. CONTRACT OR GRANT NUMBER(s) FY82 NavCompt N0002482WR1D436 Cost Code 0002R19BA463
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Element 62633N Task SF33327; RDT&E FY82
11. CONTROLLING OFFICE NAME AND ADDRESS Code R102 Naval Surface Weapons Center, White Oak Labs. Silver Spring, MD 20910		12. REPORT DATE 1 February 1982
		13. NUMBER OF PAGES 43
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Shock waves
Hydrodynamic properties
Equation of state
Sea water

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

J. M. Richardson, A. B. Arons, and R. R. Halverson (Journal of Chemical Physics, Vol. 15, 1947) developed a calculation procedure for determining the hydrodynamic properties of sea water at the front of a shock wave. The procedure has been programmed for the HP41CV, which is a hand-held programmable calculator. The program, which uses 374 lines of code, reproduces the values for a shock wave as tabulated by Richardson, et al. The advantage of the HP41CV program is that properties can be calculated without use of tables. Copies of the four magnetic cards which have the program stored can be obtained on request.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

ABSTRACT

J. M. Richardson, A. B. Arons, and R. R. Halverson (Journal of Chemical Physics, Vol. 15, 1947) developed a calculation procedure for determining the hydrodynamic properties of sea water at the front of a shock wave. The procedure has been programmed for the HP41CV, which is a hand-held programmable calculator. The program, which uses 374 lines of code, reproduces the values for a shock wave as tabulated by Richardson, et al. The advantage of the HP41CV program is that properties can be calculated without use of tables. Copies of the four magnetic cards which have the program stored can be obtained on request.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

I. INTRODUCTION

A basic building block for describing underwater explosions is the hydrodynamic properties across a shock wave in sea water. Underwater explosions were very extensively studied in World War II. Results from the World War II studies are reported in Underwater Explosion Research [1]. Richardson, Arons, and Halverson [2] discuss the calculation of hydrodynamic properties of sea water at the front of a shock wave. Background information on the calculations appears in Chapter 2 of Cole [3].

Richardson, et al., [2] used graphical techniques to interpolate thermodynamic data; the methods were crude and extremely tedious. With the advent of the modern programmable hand-held calculator, the results obtained by Richardson, et al., [2] can be obtained with ease. This report discusses a program for the HP41CV, which uses the data of Richardson, et al., [2] to calculate shock wave velocity, water velocity, specific volume, etc. Reference [2] is Appendix A.

Since World War II, research has continued in the area of underwater explosions. A more recent survey by Professor Holt [4] appears in the Annual Reviews of Fluid Mechanics. Holt [4] discusses underwater nuclear explosions as well as chemical explosions.

In passing, an interesting observation can be made concerning the level of talent working on underwater explosions in World War II. The names include G. I. Taylor, P. Bridgman, H. Bethe, J. von Neumann, and L. I. Sedov.

II. OUTLINE OF CALCULATION PROCEDURE

A. Iteration of Properties with Rankine Hugoniot Relations

The computer program has an iteration which is now discussed. The enthalpy jump across the shock wave is given by

$$\Delta H = \frac{1}{2}(p - p_0)(v + v_0) \tag{2.3}$$

The same equation numbers and the same symbols used by Richardson, et al., [2] are used here. Since $p >> p_0$, p_0 can be neglected. Also ΔH is given by

$$\Delta H = \omega + h \tag{2.20}$$

where ω is the undissipated enthalpy and h is the dissipated enthalpy. The dissipated enthalpy can be calculated two ways. First,

$$h_H = \Delta H - \omega$$

Second,

$$\Delta_p H = h_T = \int_{T_0}^{T_1} C_p dT \tag{2.9}$$

The symbol h_H, which does not appear in reference [2], denotes the value calculated from $\Delta H - \omega$. The symbol h_T is the value calculated from equation (2.9). Temperatures T_0 and T_1 are defined in reference [2]. The undissipated enthalpy is a function of specific volume, v, behind the shock wave; the function is

$$\omega = \frac{c_1^2}{n - 1}\left[\left(\frac{v_1}{v}\right)^{n-1} - 1\right] \tag{2.19}$$

The Tait equation of state relating specific volume and pressure for water is

$$p = A[S][(v_1/v)^n - 1]$$

where A[S] is a function only of entropy. Further

$$A[S] = c_1^2/nv_1 \tag{2.14}$$

Combining equations (2.3), (2.19), and (2.14) yields

$$h_H = \frac{c_1^2}{2nv_1}[(v_1/v)^n - 1](v + v_0) - \frac{c_1^2}{n-1}[(v_1/v)^{n-1} - 1] \qquad (2.21)$$

Equation (2.21) indicates that h_H is a function of v and v_1. The specific volume, v_1, is a function of T_1. By varying T_1 and v, the value of h_H varies.

When conditions across the shock wave are correct,

$$h_T(T_1) = h_H(v,T_1)$$

One varies v and T_1 until equality is achieved; since $v = v(p,T_1)$, only T_1 needs to be varied.

This is a very brief description of the iteration. The reader should refer to reference [2] for more information.

B. Temperature Behind Shock Wave

The temperature increase behind the shock wave was calculated using equation (II-4) of reference [2]. Although the equation is somewhat lengthy, the calculation is straightforward except for two quantities: $B'(t_0)$ and β_0. A cubic polynomial is given for $B(t_0)$; this polynomial was differentiated to obtain

$$B'(t_0) = \left.\frac{dB}{dt}\right)_{t=t_0}$$

The quantity β_0 is the coefficient of thermal expansion times specific volume and was evaluated using

$$\beta_0 \simeq \frac{\Delta v}{\Delta t}$$

where v is specific volume. The symbol t is used for $^{\circ}$C and T for $^{\circ}$K.

III. DESCRIPTION OF COMPUTER PROGRAM

A. *Computer Program Flow Chart*

A flow chart of the program is given in Figure 1. An iteration loop occurs for making h_H = HH = h_T = HT. The input "LOOP" is the criterion for acceptable difference between HH and HT. Since HH and HT have units of Joule/kg, "LOOP" also has units. The iteration variable is t_1, which is the adiabatic temperature in ^oC.

When an acceptable value for t_1 has been found, the program automatically goes to subroutine SHOCK. SHOCK calculates v, cm^3/gm; u, m/sec; U, m/sec; and c, m/sec.

Following SHOCK, the program moves to subroutine DEL T, which calculates the difference in temperature from behind to front of shock wave. A variety of subroutines are used by DEL T including CP, BTPRIME, G1, and D2.

B. Assignment of Storage Registers

For the program, 34 storage registers are used. The assignment of variables to the registers is given in Table I.

C. Program Listing

Appendix B is a listing of the program.

IV. OPERATION OF PROGRAM

The program has been recorded on magnetic cards and can be inserted into the HP41CV using a HP82104A card reader. The inputs to the program will now be described.

1. Turn on the HP41CV and activate USER.
2. Assign the program to a key on the key board by pressing

$\boxed{}$ ALPHA HH ALPHA, SIN

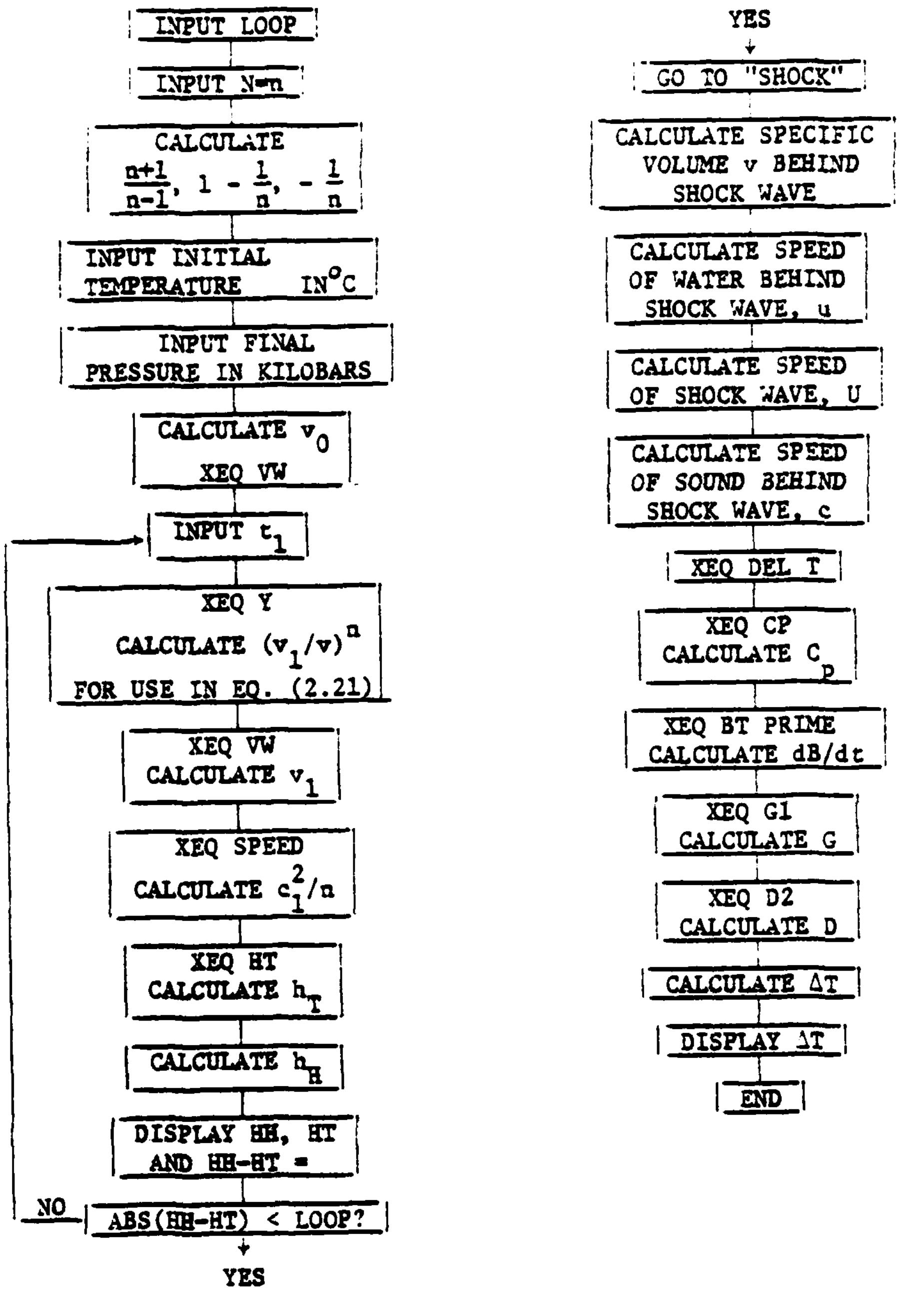


Figure 1. Computer Flow Chart.

Table I. Storage Registers

Register	Symbol	Definition	Units	Programs
01	t_1	adiabatic temperature	^{o}C	HT, VW, HH, SPEED
02	$(t-55)^2$	quantity in polynomial	^{o}C	BT
03	$(t-25)$	quantity in polynomial	^{o}C	VW
04	$(t-55)$	quantity in polynomial	^{o}C	BT
05	t_0	temperature in front of shock wave	^{o}C	HH
06	h_T	HT, dissipated enthalpy	Joule/kg	HT
07	h_H	HH, dissipated enthalpy	Joule/kg	HH
08	B(t)	quantity in Tait equation of state	kilobars	BT
09	B(t)	quantity in Tait equation of state	N/m^2	BT
10	t-55	quantity in polynomial	^{o}C	BT PRIME
11	A[S]	quantity in Tait equation of state	N/m^2	SPEED
12	p	pressure behind shock wave	kilobars	Y
13	v	specific volume	cm^3/gm	SHOCK, VW
14	u	velocity of water behind shock wave	m/sec	SHOCK
15	p	pressure behind shock wave	N/m^2	SHOCK
16	U	velocity of shock wave	m/sec	SHOCK
17	c_1	speed of sound at adiabatic temperature	m/sec	SHOCK
18	c	speed of sound behind shock wave	m/sec	SHOCK
19	β_0	coefficient of thermal expansion times specific volume v_0	$m^3/kg^{o}K$	BETA

Table I Continued. Storage Registers

Register	Symbol	Definition	Units	Programs
20	c_1^2/n	quantity in equation (2.21)	m^2/sec^2	SPEED
21	$(n+1)/(n-1)$	function of n	-	HH
22	$1-1/n$	function of n	-	HH
23	$-1/n$	function of n	-	HH
24	dB/dt	derivative of B(t)	$N/m^2\ {}^oK$	BT PRIME
25	y	$(v_1/v)^n$ ratio of specific volumes	-	HH, Y
26	v_1	specific volume at adiabatic temperature	cm^3/gm	HH, VW
27	v_0	specific volume in front of shock wave	cm^3/gm	HH
28	n	exponent in Tait equation of state	-	HH
29	C_p	heat capacity at constant pressure	Joule/kg ^oK	CP
30	G	quantity in equation (II-4)	^oK	G2, DEL T
31	D	quantity in equation (II-4)	-	D2, DEL T
32	$1+p/B$	quantity in Tait equation of state	-	HH
33	$(1+p/B)^{1/n}$	quantity in Tait equation of state	-	HH
34	LOOP	criterion for acceptable difference between HH and HT; typical value 1.0	Joule/kg	HH

In this example, the program has been assigned to the SIN key with blue H.

3. Press SIN and observe LOOP?. LOOP is the value of HH-HT which is acceptable. When ABS(HH-HT) is less than LOOP, the program is out of the iteration loop. A typical value for LOOP could be 1.0 Joule/kg. Values for a sample case are given in parentheses. (1.0)
4. Press R/S and observe N?. N is the exponent in Tait equation of state. Holt [4] uses 7.0. Richardson, <u>et al</u>., [2] use 7.15. Insert a value for N. (7.15)
5. Press R/S and observe TEMP 0?. This is the temperature in front of the shock wave in ^{o}C. Insert a value. (0)
6. Press R/S and observe PRESSURE?. This is the pressure behind the shock wave in kilobars. Insert a value. (80)
7. Press R/S and observe TEMP 1?. This is the adiabatic temperature discussed in reference [2]. Insert a value in ^{o}C. (180)
8. Press R/S and observe crows foot moving across the display. In 10 seconds, a value for HH appears. (520,245)
9. Press R/S and observe value for HT. (724,255)
10. Press R/S and observe value for HH-HT. (-204,010)
    Since the value for HH-HT exceeds LOOP, a new value for t_1 = TEMP 1 is needed.
11. Press R/S and observe TEMP 1?. To decide what value of TEMP 1 to insert, look at equation (2.9); h_T increases as T_1 or TEMP 1

increases. Although not readily apparent from equation (2.21), h_H decreases as TEMP 1 increases. For the example, a small table can be made as follows:

TEMP 1	HH	HT	HH-HT
180	520,245	724,255	- 204,010
160	569,862	642,846	- 72,984
149.2	591,574	598,988	- 7,414
148.0	593,821	594,120	- 299
147.75	594,285	593,106	1,180
147.949	593,916	593,913	3
147.9496	593,915	593,915	- 0.7

12. Press R/S and observe the following:

v = 0.6576 (cm^3/gm)

WATER U = 1,638 (m/sec)

SHOCK U = 4,891 (m/sec)

WAVE C = 6,264 (m/sec)

13. Press R/S and observe the following:

DEL T = 371.24°K

Hence the temperature behind the shock wave is

$$T = T_0 + \Delta T = 273.16 + 371.24 = 644.4^{\circ}K$$

The calculation of shock properties has now been completed. To calculate values for another set of t_0 and p, press SIN key with blue H. The computer is in USER mode.

V. SAMPLE RESULTS

A comparison can be made between results calculated with the program and values tabulated by Richardson, et al., [2]. Table II is a comparison. Most values are within 1/2 per cent.

Since Richardson, et al., [2] used graphical techniques for interpolation, the calculator values probably are more accurate. However, the thermodynamic data for water have been represented by polynomials. Errors in curve fitting are undoubtedly a few per cent. Hence, one would expect the values predicted here to be correct within a few per cent.

VI. MAGNETIC CARDS

Copies of the magnetic cards containing this program are available on request from the following address:

Distinguished Professor Allen E. Fuhs
Department of Aeronautics, Code 67
Naval Postgraduate School
Monterey, CA 93940

(408)-646-2948
AV-878-2948

VII. CALCULATION OF WATER PROPERTIES DURING ISENTROPIC EXPANSION FROM SHOCKED CONDITIONS

In order to calculate the complete flow field due to a shaped charge jet penetrating water, the calculation of temperature, internal energy, and enthalpy during isentropic expansion is necessary. Appendix C discusses two subroutines which were developed after the main program was written. The subroutines calculate internal energy and enthalpy, subroutine E-BAR, and temperature, subroutine ISEN T.

Table II. Properties of Sea Water at a Shock Front*

p kilobars	u*** m/sec		U*** m/sec		c*** m/sec		h*** Joule/gram		v*** cm^3/gm		t** °C	t_1 °C
5	258	257	1939	1930	2201	2190	6.33	6.74	0.8572	0.8593	15	1.5960
10	434	433	2306	2290	2736	2720	25.54	25.80	0.8026	0.8040	35	6.4388
20	698	698	2868	2845	3533	3510	85.16	86.55	0.7480	0.7483	73	21.4440
30	907	905	3315	3285	4152	4125	159.07	160.5	0.7185	0.7186	111	40.0832
40	1084	1080	3695	3665	4671	4640	241.56	244.0	0.6987	0.6989	150	60.6419
50	1243	1240	4031	4000	5125	5095	328.39	331.0	0.6840	0.6842	193	82.3010
60	1386	1385	4336	4300	5533	5495	417.68	419.0	0.6727	0.6728	244	104.4985
70	1518	1515	4620	4585	5909	5870	507.16	509.0	0.6639	0.6641	304	126.6660
80	1638	1635	4892	4855	6265	6225	593.79	595.5	0.6576	0.6579	371	148.0557
90	1747	1740	5159	5120	6609	6570	673.86	676.0	0.6539	0.6542	442	167.7628

*Temperature in front of shock wave, $0^{o}C$; salinity, 0.7 m NaCl; acoustic velocity ahead of shock wave, c_0, 1443 m/sec.

**Temperature behind shock wave, °C.

***Values in right-hand column are from Table IV of reference [2].

APPENDIX A

HYDRODYNAMIC PROPERTIES OF SEA WATER AT THE FRONT OF A SHOCK WAVE*

*This is reference [2] reproduced from reference [1]. The technical paper is provided as a convenience to the reader.

Reprinted from THE JOURNAL OF CHEMICAL PHYSICS, VOL. 15, No. 11, 785–794, November, 1947
Printed in U. S. A.

Hydrodynamic Properties of Sea Water at the Front of a Shock Wave*

J. M. RICHARDSON**
Baker Laboratory, Cornell University, Ithaca, N. Y.

AND

A. B. ARONS*** AND R. R. HALVERSON****
Underwater Explosives Research Laboratory, Woods Hole Oceanographic Institution, Woods Hole, Massachusetts

(Received June 2, 1947)

The Rankine-Hugoniot relations have been applied to appropriate equation-of-state data in order to calculate the propagation velocity, particle velocity, enthalpy increment, Riemann function, etc. at shock fronts of various amplitudes in sea water. One set of tables provides values over a wide pressure range (up to about 80 kilobars) and is principally intended for use in conjunction with theories of propagation of shock waves originated by underwater explosions. A second set of tables contains values which are closely spaced up to pressures of 14 kilobars. These are calculated with somewhat greater precision and are intended for use in connection with experimental measurements of particle and propagation velocities, etc.

I. INTRODUCTION

IT has long been recognized that the velocity of propagation of sound waves of finite amplitude in a fluid medium is a function of the pressure in the wave. Lamb[1] ascribes the early development of the theory to independent investigations of Earnshaw and Riemann. Qualitatively this work indicated that, since the higher pressure portions of a wave travel with greater velocity, an arbitrarily-shaped pressure pulse of finite amplitude must, during propagation, alter its shape in such a manner as to build up into a shock front. By applying the laws of conservation of mass, energy, and momentum to the transfer of matter across the shock front, Rankine and Hugoniot obtained a set of three relations among the five variables: pressure, density, particle velocity (u), shock front

* The work described in this report was performed under National Defense Research Committee Contracts OEMsr-121 with Cornell University and OEMsr-569 with the Woods Hole Oceanographic Institution.

** Present address: Bell Telephone Laboratories, Inc., Murray Hill, N. J.

*** Present address: Department of Physics, Stevens Institute of Technology, Hoboken, N. J.

**** Deceased.

[1] H. Lamb, *Hydrodynamics* (Cambridge University Press, London, 1932) 6th Ed., p. 481.

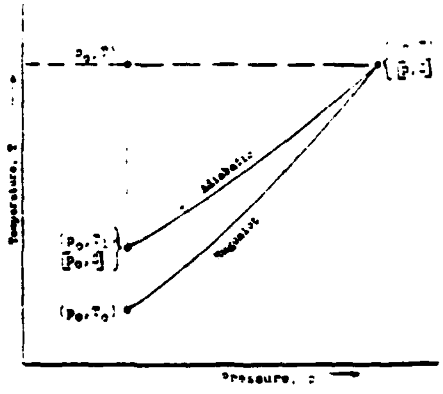


FIG. 1. Adiabatic and Hugoniot contours in $p-T$ plane.

velocity (U), and enthalpy increment (ΔH). These relations, when applied to data on the equation-of-state and specific heat, make it possible to calculate u, U, and ΔH and to evaluate certain other functions applicable to the theory of the formation and propagation of shock waves originated by explosions.[2]

Precise knowledge of u and U also makes it possible to calculate shock wave pressures in cases where the particle velocity or propagation velocity can be measured. The purpose of the calculations described below was to apply the Hugoniot relations to appropriate equation-of-state data for sea water in order to provide (a) tables of the desired functions up to very high pressures (*ca.* 80 kilobars) for use in the theory of propagation of underwater explosion waves,[2] and (b) tables of particle and propagation velocity at fairly close pressure intervals in a lower pressure region (up to *ca.* 14 kilobars).

II. OUTLINE OF THE THEORY AND COMPUTATIONAL PROCEDURES

In this section we give an account of the hydrodynamical and thermodynamical relations, and the computational procedures leading to the numerical results tabulated in Sections III and IV. For the convenience of the reader a glossary of symbols is presented in Appendix III.

When a shock wave advances with velocity U into a stationary fluid of unperturbed pressure p_0 and specific volume v_0, the pressure p, specific volume v, and particle velocity u of the fluid behind the shock front are determined by the Rankine[3]-Hugoniot[4] conditions, which express the conservation of mass, momentum, and energy of an element of fluid passing through the front. For the purposes of this paper, these conditions may conveniently be written

$$u = [(p-p_0)(v_0-v)]^{1/2}, \tag{2.1}$$

$$U = v_0[(p-p_0)/(v_0-v)]^{1/2}, \tag{2.2}$$

$$\Delta H = (1/2)(p-p_0)(v+v_0). \tag{2.3}$$

In the last equation, ΔH is the specific enthalpy increment of an element of fluid when it passes through the front. The specific enthalpy is defined as the sum of the internal energy per gram and the pressure-volume product, pv.

Given equation of state and specific heat data for the fluid, any three of the variables p, v, U and u may be determined as functions of the fourth. Here we shall regard p as the independent variable. For certain hydrodynamic applications we must have, in addition to v, U, and u as functions of p, the sound velocity

$$c = (\partial p/\partial \rho)^{1/2}_S; \quad \rho = 1/v, \tag{2.4}$$

the Riemann σ-function†

$$\sigma = \int_{p_0}^{p} [v[p', S]/c[p', S]]dp', \tag{2.5}$$

and the undissipated enthalpy

$$\omega = \int_{p_0}^{p} v[p', S]dp', \tag{2.6}$$

where S is the entropy.

In practice, one must resort to successive approximations to effect a reduction of the Hugoniot conditions, combined with equation-of-state and specific heat data, to a set of relations expressing u, U, and v as functions of p. To this

[2] J. G. Kirkwood and H. Bethe, *The Pressure Wave Produced by an Underwater Explosion* (Dept. of Commerce Bibliography No. PB 32182), OSRD Report No. 588, Part I.

[3] W. J. M. Rankine, Trans. Roy. Soc. London, A160, 277 (1870).

[4] H. Hugoniot, J. de l'ecole polyt. 51, 3 (1887); 58, 1 (1888).

† The Riemann σ-function occurs in Riemann's form of the hydrodynamical equations, which, for the case of spherical symmetry, may be written (see reference 1):

$$[(\partial/\partial t)+(c+u)(\partial/\partial r)](\sigma+u) = -2cu/r,$$
$$[(\partial/\partial t)-(c-u)(\partial/\partial r)](\sigma-u) = 0,$$

where t is the time and r is the radial coordinate. The other quantities have already been defined.

end, it is expedient first to consider certain quantities as functions of pressure and temperature, p and T, or pressure and entropy, p and S. Before proceeding to a more detailed discussion of the calculations it may perhaps help to orient the reader if we consider, qualitatively, contours of some pertinent quantities in the p–T plane. In Fig. 1, the possible states of a given fluid just behind the shock front lie along a single curve, which we have labeled "Hugoniot." An element of fluid initially in the state (p_0, T_0) which has attained a state (p, T) just behind the shock front finally returns to a state (p_0, T_1) along the adiabatic, so labeled in the figure. Also included are the designations of a few points on a p–S basis using square brackets according to the convention introduced in Part c of this section. In general, T_1 is larger than T_0 because of the dissipation occuring at the front. The central part of our problem is the determination of the Hugoniot curve.

We shall consider in Part a the calculations due to Arons and Halverson[5] which are intended to be accurate in the range of relatively low pressure (*ca.* 0 to 20 kilobars). These results as stated in the introduction are intended for the determination of the peak pressure of a shock wave from measured values of the shock front velocity U or particle velocity u. In Part b, we shall consider the calculations of Kirkwood and Richardson,[6] the results of which were originally intended for the applications of the shock wave propagation theory of Kirkwood and Bethe[3] which required data over a higher range of pressures (*ca.* 20 to 50 kilobars).

a. Calculations of Arons and Halverson

Here we outline the calculations[7] suitable for the relatively low pressure range (*ca.* 0 to 20 kilobars) based upon the equation-of-state and specific heat data discussed in detail in Appendix I. For the range 0 to 1.5 kilobars, the Ekman equation-of-state was used; in the range 0 to 25 kilobars, the Tait equation-of-state,

$$[v(0,T)-v(p,T)]/v(0,T)=(1/n)\log[1+p/B(t)],\quad t=(T-273.16)°\text{C}. \tag{2.7}$$

In the first case, the initial temperature was $t_0=15$°C; in the second, $t_0=25$°C. In both cases, the initial pressure $p_0=0$. Neither of the two equations-of-state are complete in the sense that $v_0=v(0,T)$ must be determined by auxiliary thermal expansion data (also discussed in Appendix I).

We express the enthalpy and volume increments

$$\Delta H=H(p,T)-H(0,T_0),\qquad \Delta v=v(p,T)-v(0,T_0), \tag{2.8}$$

in terms of line integrals, first along an isobar from $(0,T_0)$ to $(0,T)$ and, secondly, along an isotherm from $(0,T)$ to (p,T) (see Fig. 1). For the enthalpy increment we obtain

$$\begin{aligned}\Delta H&=\Delta_pH+\Delta_TH,\\ \Delta_pH&=\int_{T_0}^{T}C_p(0,T')dT'=\bar{c}_p\Delta T,\\ \Delta_TH&=\int_0^p\left[v(p',T)-T\frac{\partial v(p',T)}{\partial T}\right]dp',\\ \Delta T&=T-T_0,\end{aligned} \tag{2.9}$$

where $c_p(0,T)$ is the specific heat extrapolated to zero pressure and $\bar{c}_p$ is the mean of c_p over the temperature range ΔT.

For the volume increment we obtain

$$\begin{aligned}\Delta v&=\Delta_pv+\Delta_Tv,\\ \Delta_pv&=v(0,T)-v(0,T_0)=\bar{\beta}_0\Delta T,\\ \Delta_Tv&=v(p,T)-v(0,T),\end{aligned} \tag{2.10}$$

where $\bar{\beta}_0$ is the mean thermal expansion at zero pressure over the temperature range ΔT.

From the last Hugoniot condition, Eq. (2.3), and Eq. (2.9) we obtain

$$\Delta T=\frac{[v(0,T_0)+(1/2)(\Delta_Tv)]p-\Delta_TH}{\bar{c}_p-(1/2)(\bar{\beta}_0p)}, \tag{2.11}$$

where Δ_TH is to be calculated by means of the third of Eq. (2.9) and the appropriate equation-of-state, and where Δ_Tv is to be obtained from compressibility data. The right-hand side of Eq. (2.11) depends, of course, on the temperature T. The determination of ΔT is accomplished by the method of successive approximations. A trial

[5] A. B. Arons and R. R. Halverson, *Hugoniot Calculations for Sea Water at the Shock Front*, OSRD Report No. 6577, NDRC No. A-469.

[6] J. G. Kirkwood and J. M. Richardson, *The Pressure Wave Produced by an Underwater Explosion, Part III*, OSRD Report No. 813 (Dept. of Commerce Bibliography No. PB 32184).

[7] J. G. Kirkwood and E. Montroll, *Pressure Wave Produced by an Underwater Explosion, II*, OSRD Report No. 676 (Dept. of Commerce Bibliography PB-32183).

TABLE I. $U-c_0/c_0$, u, and ΔH in low pressure region (based on Ekman equation-of-state). Sea water: Initial temperature 15°C; salinity 32 parts per thousand (3.79 wt. percent NaCl); $c_0=4922.8$ ft/sec $=1500.5$ m/sec.

A: $p-p_0$ (kbar)	A: $\frac{U-c_0}{c_0}$ (%)	A: u (m/sec)	A: ΔH (cal/gm)	B: $p-p_0$ (lb/in²)	B: $\frac{U-c_0}{c_0}$ (%)	B: u (ft/sec)	B: ΔH (cal/gm)
0.00	0.00	0.0	0.0	0	0.00	0.0	0.0
.25	2.07	15.1	5.8	2.000	1.14	29.1	3.2
.50	4.03	31.2	11.5	4.000	2.27	57.6	6.4
.75	5.93	46.2	17.2	6.000	3.35	85.5	9.6
1.00	7.81	60.5	23.0	8.000	4.43	112.9	12.7
1.25	9.65	74.4	28.5	10.000	5.48	139.7	15.9
1.50	11.44	87.7	34.1	12.000	6.53	166.0	19.0
				14.000	7.56	191.8	22.1
				16.000	8.58	217.1	25.2
				18.000	9.59	242.0	28.3
				20.000	10.57	266.5	31.4
				22.000	11.55	290.6	34.5

TABLE II. $U-c_0/c_0$, u, and ΔH in intermediate pressure region (1.5 to 25 kilobars) (based on Tait equation-of-state; $n=7.800$, $B=3.012$). Sea water: Initial temperature 25°C; salinity 32 parts per thousand (3.79 wt. percent NaCl): $c_0=5014.7$ ft/sec $=1528.5$ m/sec.

A: $p-p_0$ (kbar)	A: $\frac{U-c_0}{c_0}$ (%)	A: u (m/sec)	A: ΔH (cal/gm)	B: $p-p_0$ (lb/in²)	B: $\frac{U-c_0}{c_0}$ (%)	B: u (ft/sec)	B: ΔH (cal/gm)
0.0	0.00	0.0	0.0	0	0.0	0	0.0
1.0	8.18	59.2	23.0	20.000	11.0	258	31.3
1.5	11.58	85.5	34.2	30.000	15.8	375	46.6
2.0	15.39	110.1	45.3	40.000	20.5	483	62.0
2.5	18.71	135.0	56.3	50.000	24.7	584	77.0
3.0	21.88	157.8	67.2	60.000	28.7	676	92.0
4.0	27.84	200.5	88.8	70.000	32.5	767	107.0
5.0	33.39	240.2	110.1	80.000	36.0	853	121.0
6.0	38.59	277.5	131.2	90.000	39.4	937	135.0
8.0	48.13	346.1	172.9	100.000	42.8	1014	150.0
10.0	56.73	408.9	214.0	120.000	49.2	1168	178.0
12.0	64.70	466.9	254.8	140.000	55.2	1312	207.0
14.0	72.25	510.9	295.2	160.000	60.8	1445	235.0
25.0	108.40	769.1	514.4	180.000	66.3	1564	263.0
				200.000	71.5	1662	291.0

value of ΔT is used in evaluating the right-hand side giving a more accurate value of ΔT on the left-hand side, and the process is repeated until the results of two successive steps differ by a sufficiently small amount. One or two steps generally suffice.

We have thus obtained T as a function of p along the Hugoniot curve (see Fig. 1). It is now possible to calculate immediately the particle velocity u, the propagation velocity U, and the specific volume v as functions of p behind the shock front. The results using the Ekman equation-of-state and the Tait equation-of-state are tabulated in Tables I and II, respectively, of Section III.

b. The Calculations of Richardson and Kirkwood

Here we outline the calculations[6] intended for the applications of the shock wave propagation theory of Kirkwood and Bethe.[2] These are based upon the equation-of-state and specific heat data discussed in detail in Appendix II. We use a modified Tait equation-of-state connecting $v(p, T)$ and $v(0, T)$ to be discussed below. In most respects, the data is made to fit the properties of an aqueous 0.7 molal NaCl solution assumed to be roughly equivalent to sea water of salinity $s=32$ parts per thousand (see Section 1 of Appendix I).

In these calculations the initial pressure p_0 is taken to be zero, and several different initial temperatures T_0 are used: 0°C, 20°C, and 40°C.

Before indicating the precise nature of the modification of the Tait equation, it is desirable to mention that in this part two different pairs of independent variables will be used: pressure and temperature (p, T), and pressure and entropy $[p, S]$. Consequently, in order to indicate which pair are used in a function, we will use parenthesis to indicate the first pair and square brackets to indicate the second, i.e. $v(p, T)$ and $v[p, S]$.

The modified form of Tait equation introduced by Kirkwood[2,6] is

$$\log(v_1/v)=(1/n)\log(1+p/A[S]), \quad (2.12)$$

where

$$v=v[p, S]=v(p, T[p, S]), \quad v_1=v[0, S],$$

(see Fig. 1) n is an empirical constant, and the function $A[S]$ is related to the function $B(t)$ in the original isothermal form of the Tait equation, Eq. (2.7), as follows,

$$A[S]=B(t[0, S]), \quad t=(T-273.16)\text{°C}. \quad (2.13)$$

The reasons for introducing this modification of the Tait equation are at least twofold: (1) the anomaly of a vanishing specific volume $v(p, T)$ at a finite pressure along a given adiabatic (which does not differ markedly from the Hugoniot curve in the case of water) is removed to a higher pressure by replacing $[v(0, T)-v(p, T)]/[v(0, T)]$

TABLE III. Values of $U-c_0/c_0$ for different temperatures and salinities at a shock wave peak pressure of 1.00 kilobar.

Salinity (parts per 1000)	Temperature (°C)	$U-c_0/c_0$ (%)
32	15	7.81
32	25	7.81
35	15	7.76

by $\log(v[0, S]/v[p, S])$, and (2) the calculation of quantities defined by line integrals along adiabatics is greatly simplified by taking S instead of T as one of the independent variables.

The function $A[S]$ is related simply to c_1, the sound velocity at zero pressure and entropy S according to Eq. (2.4) as follows

$$A[S] = c_1^2/nv_1; \quad c_1 = c[0, S]. \tag{2.14}$$

On the basis of Bridgman's p-v-T data for pure water, an average value of n equal to 7.15 has been selected for the present calculations. In Section 2 of Appendix II, it is shown that n deviates from this value by less than 4 percent in a large pressure-temperature field bounded by adiabatics starting at zero pressure and temperatures of 20°C and 60°C, respectively, and extending to pressures of 25,000 kg/cm². We assume that n has the same value for an aqueous 0.7 molal NaCl solution as for pure water, and we obtain by interpolation the required values of $B(t)$ from R. E. Gibson's values of $B(t)$ for dilute aqueous NaCl solutions (see Appendix II, Section 1). The appropriate heat capacity and thermal expansion data are discussed in Section 1 of Appendix II.

We now proceed to the calculation of the quantities u, U, c, σ, and ω. We first express these quantities with use of Eq. (2.12) in terms of p,

$$v = v(p, T) = v[p, S], \quad v_0 = v(0, T_0),$$

$v_1 = v(0, T_1) = v[0, S]$, and $c_1 = c(0, T_1) = c[0, S]$ (see Eqs. (2.1)–(2.6), also Fig. 1) as follows:

$$u = [p(v_0 - v)]^{\frac{1}{2}}, \tag{2.15}$$

$$U = pv_0/u, \tag{2.16}$$

$$c = c_1(v_1/v)^{(n-1)/2}, \tag{2.17}$$

$$\sigma = \frac{2c_1}{n-1}[(v_1/v)^{(n-1)/2} - 1], \tag{2.18}$$

$$\omega = \frac{c_1^2}{n-1}[(v_1/v)^{n-1} - 1]. \tag{2.19}$$

Once the temperature T_1, to which an element of fluid returns along the adiabatic intersecting the Hugoniot curve at (p, T), is determined, all of the above quantities may be determined as functions of p. To accomplish this, the enthalpy increment, ΔH, occurring in the third Hugoniot condition, Eq. (2.3), is written as the sum of two line integrals, the first along an isobar from $(0, T_0) = [0, S_0]$ to $(0, T_1) = [0, S]$ and the second along an adiabatic from $(0, T_1) = [0, S]$ to $(p, T) = [p, S]$ (see Fig. 1), giving:

$$\Delta H = \omega + h,$$

$$\omega = \int_0^p v[p', S]dp', \tag{2.20}$$

$$h = \int_{S_0}^{S} T[0, S']dS' = \int_{T_0}^{T_1} c_p(0, T')dT',$$

where ω is the undissipated enthalpy already defined by Eq. (2.6) with $p_0 = 0$ and given explicitly in terms of c_1, v_1, and v in Eq. (2.19). The dissipated enthalpy h can be determined as an explicit function of T_0 and T_1 from specific heat data (Appendix II, Section 1). Combining the third Hugoniot condition, Eq. (2.3), with Eqs. (2.19) and (2.20) we obtain the relation

$$\frac{h}{c_1^2} = \frac{1}{2n}\left[y - \frac{n+1}{n-1}(y^{(1-1/n)} - 1) - y^{-1/n}\right] - \frac{v_1 - v_0}{2nv_1}(y - 1), \tag{2.21}$$

TABLE IV. Properties of sea water at a shock front. (Initial temperature 0°C; salinity 0.7 m NaCl; $C_0 = 1443$ m/sec.)

p (kilo-bar)	u (m/sec)	U (m/sec)	c (m/sec)	σ (m/sec)	$\omega \times 10^{-6}$ (m/sec)²	h (joule/gm)	v (cm³/gm)
0	0	—	—	0	0	0	0.9915
5	257.0	1930	2190	253.5	0.4566	6.740	.8593
10	433.0	2290	2720	420.5	0.8720	25.80	.8040
15	575.0	2585	3145	552.0	1.270	54.40	.7710
20	697.5	2845	3510	664.0	1.655	56.55	.7483
25	805.5	3075	3835	763.5	2.030	122.5	.7319
30	905.0	3285	4125	855.5	2.405	160.5	.7186
35	997.0	3480	4395	940.5	2.770	201.5	.7075
40	1080	3665	4640	1020	3.140	244.0	.6989
50	1240	4000	5095	1175	3.860	331.0	.6842
60	1385	4300	5495	1315	4.575	419.0	.6728
70	1515	4585	5870	1455	5.285	509.0	.6641
80	1635	4855	6225	1585	6.000	595.5	.6579
90	1740	5120	6570	1705	6.730	676.0	.6542

TABLE V. Properties of sea water at a shock front. (Initial temperature 20°C; salinity 0.7 m NaCl; $C_0 = 1517$ m/sec.)

p (kilo-bar)	u (m/sec)	U (m/sec)	c (m/sec)	σ (m/sec)	$\omega \times 10^{-6}$ (m/sec)²	h (joule/gm)	v (cm³/gm)
0	0	—	—	0	0	0	0.9929
5	251.0	1975	2230	248.5	0.4595	5.570	.8668
10	425.5	2335	2755	415.5	0.8790	23.45	.8120
15	567.0	2630	3175	549.0	1.280	49.35	.7787
20	689.0	2880	3535	663.0	1.670	90.05	.7555
25	798.0	3110	3855	765.0	2.050	115.0	.7381
30	897.5	3320	4140	859.0	2.425	152.5	.7243
35	990.0	3510	4405	946.5	2.795	192.0	.7130
40	1075	3690	4480	1030	3.160	233.0	.7034
50	1235	4020	5100	1185	3.885	317.5	.6880
60	1380	4325	5305	1330	4.605	404.5	.6763
70	1510	4610	5880	1465	5.320	489.5	.6679
80	1625	4885	6240	1600	6.050	573.5	.6626

TABLE VI. Properties of sea water at a shock front. (Initial temperature 40°C; salinity 0.7 m NaCl.)

p (kilobar)	u (m/sec)	U (m/sec)	c (m/sec)	σ (m/sec)	$\omega \times 10^{-6}$ (m/sec)^2	h (joule/gm)	v (cm^3/gm)
0	0	—	—	0	0	0	0.9993
5	249.5	2005	2255	247.0	0.4630	5.59	.8749
10	423.5	2360	2775	415.0	0.8870	22.75	.8198
15	566.5	2645	3195	550.0	1.290	48.30	.7859
20	689.0	2900	3550	666.5	1.685	78.70	.7621
25	798.5	3130	3865	770.5	2.065	113.5	.7441
30	899.0	3335	4150	866.0	2.445	151.0	.7298
35	992.0	3525	4415	955.5	2.815	189.5	.7179
40	1080	3705	4660	1040	3.185	230.5	.7080
50	1240	4035	5110	1200	3.915	315.0	.6926
60	1380	4340	5515	1345	4.640	400.5	.6813
70	1510	4635	5900	1485	5.370	483.0	.6737

where $y = (v_1/v)^n$. With the aid of tables of n/c_1^2 and v_1 as functions of T_1 and T_0, Eq. (2.21) may be solved by successive approximations giving T_1 as a function of the parameter y. Since the equation of state, Eq. (2.12), may be expressed simply as $p = B(T_1 - 273.16)[y-1]$, the temperature T_1 may be determined as a function of the pressure p by a tabular elimination of y. By graphical interpolation, T_1 is finally determined for the desired integral values of p (in kilobars), and the functions u, U, c, σ, and ω are then computed as functions of p by means of Eqs. (2.15)–(2.19).

III. NUMERICAL RESULTS OF ARONS AND HALVERSON

In fundamental shock wave studies, it is frequently necessary to know values of $U - c_0/c_0$ and u at given pressure levels to the highest possible degree of accuracy. With this object in view, the calculation methods described in Section 2A were applied to the best available equation-to-state data. The numerical results are given in Tables I and II. A critical discussion of the equation-of-state data will be found in Appendix I together with references to the sources from which they were obtained.

Table I gives results for the "low pressure" region, covering shock wave peak pressures of from 0 to 1.50 kilobars (*ca.* 22,000 p.s.i.). The calculations in this table were based upon the Ekman equation-of-state for sea water (see Appendix I) which is used in the calculation of sound velocity for echo-ranging tables.

Since the Ekman equation deviates appreciably from experimental compressibility data at pressures exceeding 2 kilobars, this equation was abandoned in the "intermediate pressure" region. The results in Table II are applicable principally to the region between 1.5 and 14 kilobars (*ca.* 200,000 p.s.i.) and are based on a careful fit of the Tait equation to Adams's experimental compressibility data (see Appendix I).

Tables I and II were computed for certain specific values of temperature and sea water salinity (equivalent to 0.675 molal NaCl), and it is shown in Table III that the value of $U - c_0/c_0$ is not very sensitive to changes in these variables.

IV. NUMERICAL RESULTS OF KIRKWOOD AND RICHARDSON

In Tables IV to VI, the particle velocity u, the shock front velocity U, the sound velocity c, the Riemann σ-function, the undissipated enthalpy ω, the dissipated enthalpy h, and the specific volume v of sea water (0.7 molal NaCl solution) are presented as functions of pressure p along three Hugoniot curves, starting at zero pressure and the temperatures 0°C, 20°C and 40°C, respectively. These results have been calculated by the procedures of Part b of Section 2 and the data of Appendix II. The results above 30 kilobars represent extrapolations beyond the range of experimental data; consequently the validity of the results above, say, 50 kilobars, is questionable.

In closing this discussion of the calculations, the authors wish to acknowledge their gratitude and appreciation to Professor J. G. Kirkwood of Cornell University for his contributions in initiating the work and in supplying valuable guidance and advice.

APPENDIX I††

1. Salinity and Temperature Conditions

All calculations were made for sea water having a salinity of 32 parts per thousand (the average salinity of sea water at Woods Hole, Massachusetts). Salinity is defined in terms of directly measured chlorinity as:

$$s = 0.030 + 1.8050 \text{ Cl}$$

where s and Cl are expressed in parts per thousand.

It was calculated from the average composition

†† Equation-of-state data used in computation of Tables I and II.

of sea water that (on the basis of *ionic strength*) a salinity of 32 parts per thousand is equivalent to an NaCl solution having a molality of 0.675 or a weight percentage of 3.79 percent NaCl.

Table I was computed for an initial temperature of 15°C because this temperature is a rough average of conditions normally encountered in experimental work. Table II was computed for an initial temperature of 25°C because this was the temperature quoted for the available compressibility data.[8] Table III shows that the results are not sensitive to small variations in temperature and salinity.

2. Specific Volume and Coefficient and Thermal Expansion

The best sources of data seem to be the oceanographical tables of Knudsen.[9] Second power equations in t(°C) were fitted to the data tabulated for $s=32$:

For Table I:

$$v(t)=0.97709+2.05\times10^{-4}(t-15)+4\times10^{-6}(t-15)^2.$$

For Table II:

$$v(t)=0.97956+2.85\times10^{-4}(t-25)+4\times10^{-6}(t-25)^2.$$

3. Heat Capacity

The heat capacity data used in computing Tables I and II are those quoted by S. Kuwahara:[10]

$$C_p=C_{p'}-0.0004226t+0.0000063216t^2\ \text{cal./gm°C},$$

TABLE VII. Comparison of experimentally measured sound velocity with calculations based on the Ekman Compressibility Equation. (Salinity = 31.7 parts per thousand.)

Temperature (°C)	Velocity of sound (ft/sec) Measured	Calculated	Deviation (%)
10.9	4887.7	4875.6	0.26
11.6	4885.8	4883.8	0.04
11.6	4893.1	4883.8	0.19
11.5	4902.4	4882.5	0.41
11.1	4888.3	4878.5	0.20

[8] Adams, J. Am. Chem. Soc. 53, 3769 (1931).

[9] *Oceanographical Tables*, Comissariat of Agriculture, USSR, Moscow, 1931. (A general compilation of oceanographic data by N. N. Zubov.)

[10] S. Kuwahara, *Velocity of Sound in Sea Water and Calculation of the Velocity for Use in Sonic Sounding* (Hydrographic Dept. I.J.N. Tokyo, 1938).

TABLE VIII. Comparison of Adams's experimental compressibilities and the empirical fit given by the Ekman and Tait equations.

p (kbar)	$(v_0-v)/v_0$: Adams (experimental) Pure H_2O	Adams (experimental) 3.79% NaCl	Ekman equation $S=32$ (Table I)	Tait* equation $n=7.800$ $B=3.012$ Table II)	Tait* equation $n=7.445$ $B=3.156$
0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.5	.0212	.0196	.0198	.0197	.0198
1.0	.0393	.0368	.0370	.0368	.0370
1.5	.0555	.0522	.0522	.0518	.0522
2.0	.0699	.0638	.0655	.0653	.0659
3.0	.0945	.0894	.0871	.0887	.0897
4.0	.1152	.1091	—	.1083	.1095
5.0	.1330	.1265	—	.1254	.1275
6.0	.1485	.1417	—	.1405	.1431
7.0	.1622	.1552	—	.1540	.1569
8.0	.1746	.1670	—	.1662	.1695
9.0	.1858	.1781	—	.1775	.1812
10.0	.1964	.1886	—	.1876	.1917
11.0	.2059	.1980	—	.1972	.2017

* $(v_0-v)/v_0=(1/n)\log(1+p/B)$.

where

$$C_{p'}=1.005-0.004136s+0.0001098s^2-0.000001324s^3$$

In the above equations, t is temperature in °C and s is salinity in parts per 1000. These data are in good agreement with those used by Kirkwood and Richardson, quote in Appendix II.

4. Compressibility Data for Low Pressure Region (Table I)

The following equation was used in computing Table I:

$$10^6\mu=\frac{4886}{1+0.183p}-[227+28.33t-0.551t^2+0.004t^3]+p[105.5+9.50t-0.158t^2]-1.5p^2t-\left(\frac{\gamma-28}{10}\right)[147.3-2.72t+0.04t^2-p(32.4-0.87t+0.02t^2)]+\left(\frac{\gamma-28}{10}\right)^2[4.5-0.1t-p(1.8-0.06t)],$$

where p is pressure in kilobars, t is temperature in degrees centigrade, and μ is defined by:

$$v=v_0(1-\mu p),$$

γ is defined by:

$$\gamma=-0.069+1.4708\ \text{Cl}-0.001570\ \text{Cl}^2+0.0000398\ \text{Cl}^3.$$

The above empirical equation for sea water compressibility is due to Ekman[11] and has been widely used for computation of sound velocity in sea water.[10,12] The validity of the Ekman equation for sound velocity calculations was verified experimentally as indicated in Table VII.

Experimental sound velocity measurements were made by recording with a rotating drum camera the signals applied to a cathode ray oscilloscope by two very small piezoelectric gauges placed a known distance apart. The sound source was a No. 8 detonator cap placed far enough away from the gauges so that the effect of finite pressure amplitude was less than 0.03 percent. An error of about 0.2 percent was inherent in the experimental work owing to slight errors in the alignment of the two recording gauges with the sound source. This accounts for the magnitude and systematic nature of the discrepancy apparent in Table VII.

Further verification of the applicability of Ekman's equation in the region up to 1.50 kilobars is given in Table VIII where values obtained from the equation are compared with the experimental values of Adams for NaCl solutions.

5. Compressibility Data for Intermediate Pressure Region (Table II)

As indicated in Part 1 of this appendix, a sea water salinity of 32 parts per thousand corresponds to a 3.79 weight percent solution of NaCl. The compressibility of NaCl solution of this concentration was obtained by graphical interpolation of Adams's data.[4]

The Tait equation in the form:

$$v(0,T)-v(p,T)/v(0,T)=(1/n)\log[1+p/B(t)],$$
$$t=(T-273.16)^{\circ}\mathrm{C}$$

was then fitted to Adams's data. In an effort to check the Tait equation against the Ekman equation used for computation of Table I, a fit was first made to the lower pressure region. Values of n and $B(25^{\circ}\mathrm{C})$ were so selected that the equation not only fitted the data of Adams with adequate precision but also yielded the correct velocity of sound in the limit of zero pressure. This additional restriction (that the equation give $c_0=1528$ m/sec. at 25°C and $s=32$) required that $nB(25^{\circ}\mathrm{C})=23.497$, the latter relation being obtained from the thermodynamic equations:

$$\left(\frac{\partial v}{\partial p}\right)_T=-\frac{v_0^2}{c_0^2}-\frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p^2,$$
$$\left(\frac{\partial v}{\partial p}\right)_T=-\frac{v_0}{nB(t)}\text{ at }p=0.$$

In this case n was taken as 7.445 and $B(25^{\circ}\mathrm{C})$ as 3.156 kilobars, and the resulting equation fits the data of Adams quite closely up to pressures of about 4 kilobars as shown in Table VIII. For purposes of further calculation, the temperature *variation* of B was assumed to be the same as that used by Kirkwood and Richardson on the basis of a private communication from Gibson (see Appendix II). Calculation of $U-c_0/c_0$ at 1.00 kilobar yielded a value of 7.85 percent, in good agreement with the value of 7.81 percent obtained from the Ekman equation.

Having verified the accuracy of results obtained from the Tait equation when fitted as described above, the same technique was used to fit the equation to the intermediate pressure range (up to values for 11 kilobars quoted by Adams; it was assumed safe to extrapolate the resulting equation to pressures of 14 or 15 kilobars). It was found that the best fit of the data as well as a correct value for the velocity of sound were obtained by taking $n=7.300$ and $B(25^{\circ}\mathrm{C})=3.012$, the temperature variation of B again being assumed to be that mentioned above. The Tait equation containing these parameters was then used for the computation of Table II. The fit of the equation to Adams's data is shown in Table VIII.

TABLE IX. Values of n computed from $p-v-T$ data (using $n=7.15$ in computation of ΔT).

t_0^a (°C)	p (kg/cm²) 5000	15,000	25,000
20	7.211	7.183	7.130
40	7.360	7.126	6.969
60	7.411	7.054	6.868

[a] t_0 = centigrade temperature through which the adiabatic for S passes at zero pressure.

[11] V. W. Ekman, *Publications de Circonstance No. 43* (Conceil Permanent Internationale Pour L'Exploration de la Mer, November 1908).

[12] Matthews, *Tables of the Velocity of Sound in Pure Water and Sea Water for Use in Echo Sounding and Sound Ranging* (Hydrographic Dept., Admiralty, H.D. No. 282).

APPENDIX II

1. Data Employed in the Computations of Part B of Section II

In the modified Tait equation, Eq. (2.12), the function $A[S]=B(t)$, where $t=T[0,S]-273.16$,

is determined from the empirical values of $B(t)$, fitting the original isothermal Tait equation, Eq. (2.7), to experimental data. R. E. Gibson[13] gives third-degree t-expansions of $B(t)$ for various molalities of NaCl. By interpolating the coefficients (the constant term numerically and the other graphically) for a molality of 0.7, one obtains $B(t)=3.134-1.65\times10^{-2}(t-55)-1.181\times10^{-4}(t-55)^2+5.32\times10^{-7}(t-55)^3$ kilobars.

The specific heat $c_p(0, T)$ for a 0.7 molal NaCl solution was obtained by interpolation from the values quoted in the *International Critical Tables* and *Physikalischchemische Tabellen*. The resulting set of values is fitted adequately by the expression

$$c_p(0, t+273.16)=3.9644+6.24\times10^{-4}t \text{ joule/gm. deg.}$$

From Gibson and Loeffler[14] a set of values of $v(0, T)$ covering the range from 25°C to 95°C inclusive was obtained for a 0.7 molal NaCl solution by means of empirical equations giving $v(0, T)$ as a function of concentration for each temperature. In extrapolating to higher temperatures, the relation,

$$v(0, t+273.16)=0.994150+2.929\times10^{-4}(t-25)+3.241\times10^{-6}(t-25)^2 \text{ cm}^3/\text{gm.}$$

was used; for lower temperatures ($t<10$°C),

$$v(0, t+273.16)=0.991442+6.025\times10^{-6}(t-3.8)^2 \text{ cm}^3/\text{gm.}$$

2. Test of the Modified Tait Equation with Bridgman's Data for Pure Water. Determination of the Characteristic Constant n.

The modified Tait equation-of-state, Eq. (2.12), may for our present purposes be written in the form

$$\log(v[0, S]/v[p, S]) = (1/n)\log(1+p/A[S]) \qquad \text{(II-1)}$$

where $A[S]$ is related to the $B(t)$ in the original isothermal equation of state as follows,

$$A[S]=B(t_0),\quad t_0=T_0-273.16,\quad T_0=T[0, S]. \qquad \text{(II-2)}$$

According to the convention introduced in Part B of Section II, parentheses ( ) after a function denotes that the independent variables are p and T, whereas square brackets [ ] denote that they are p and S.

Now we wish to test Eq. (II-1) with Bridgman's[15] p–v–T data for pure water with the ultimate object of finding the best value for n. We assume implicitly that n does not vary rapidly with NaCl concentration. To make the comparison, we first must know the values of the temperature T corresponding to the various points $[p, S]$, the calculation of which we consider below.

Letting $T[0, S]=T_0$, $T[p, S]=T$, and $T-T_0=\Delta T$, we have

$$\Delta T=\int_0^p \frac{\partial T[p, S]}{\partial p}dp=\int_0^p\frac{\partial v[p, S]}{\partial S}dp. \qquad \text{(II-3)}$$

Using Eq. (II-1), a simple calculation yields

$$\Delta T=\frac{G}{(1+p/A)^{1/n}}[(1+D)(1+p/A)-(n+D)(1+p/A)^{1/n}+n-1], \qquad \text{(II-4)}$$

where

$$A=A[S]=B(t_0),$$

$$G=\frac{A'[S]v[0, S]}{n-1}=\frac{T_0B'(t_0)\cdot v(0, T_0)}{(n-1)\cdot c_p(0, T_0)},$$

$$D=\frac{nA[S]\dfrac{\partial v[0, S]}{\partial S}}{A'[S]v[0, S]}=\frac{nB(t_0)\beta_0}{B'(t_0)v(0, T_0)},$$

$$\beta_0=\left(\frac{\partial v(0, T)}{\partial T}\right)_{T=T_0}.$$

To calculate T, given a specified p and $T_0=T[0, S]$, a tentative value of $n=7.15$ was chosen for use in Eq. (II-4). The corresponding value of $v[p, S]=v(p, T)$ was obtained by interpolation from Bridgman's[15] p–v–T data. Inserting these values of $v[p, S]$ in Eq. (II-1), and knowing the values of $v[0, S]=v(0, T_0)$ and $B(t_0)$ for pure water, a set of values of n was calculated for $p=5{,}000$, 15,000, 25,000 kg/cm²

[13] Private communication.

[14] Gibson and Loeffler, J. Am. Chem. Soc. 53, 443 (1941).

[15] Bridgman, J. Chem. Phys. 3, 597 (1935) and private communication.

and $t_1 = T_1 - 273.16 = 20°, 40°, 60°\text{C}$, the results summarized in Table IX.

Additional data (for pure water) used in Eqs. (II-1) and (II-4) were

$$B(t) = 2.996 + 7.285 \times 10^{-3}(t-25) - 1.790 \times 10^{-4}(t-25)^2 + 6.13 \times 10^{-7}(t-25)^3 \text{ kilobars,}^{13}$$

and

$$\frac{1}{2.303}\frac{\partial \log v(0, 273.16+t)}{\partial t} = \frac{2(t-3.98)}{244{,}860 + 15{,}040(t-3.98)^{0.62}} - \frac{(0.62)(15{,}040)(t-3.98)^{1.62}}{[244{,}860+15{,}040(t-3.98)^{0.62}]^2}$$

obtained from Ipatov's[16] empirical equation for v by differentiation.

The average value of n is 7.146. In the present calculations this value has been rounded off to 7.15.

The entries in Tables IV, V, and VI therefore contain more significant figures than the test justifies. On the basis of the test, the errors associated with the use of the modified Tait equation are of the order of several percent. In particular, the results obtained for low pressures will disagree with known data by several percent.

[16] I. V. Ipatov, J. Phys. Chem. (U.S.S.R.) 5, 1230 (1934).

APPENDIX III

Symbols

$A[S]$ = parameter in modified "adiabatic" Tait equation-of-state.
$B(t)$ = parameter in isothermal Tait equation-of-state.
c = local velocity of sound.
c_0 = velocity of sound at zero pressure.
c_p = specific heat at constant pressure.
h = dissipated enthalpy increment: $\int_{T_0}^{T} c_p(0, T')dT'$.
ΔH = enthalpy increment: $\Delta H = \omega + h$.
n = characteristic constant in Tait equation-of-state.
p_0 = initial pressure ahead of shock front, $p_0 = 0$, in these calculations.
p = pressure behind shock front.
S = entropy.
s = sea water salinity.
t = temperature in °C.
T = absolute temperature.
u = particle velocity behind shock front.
U = shock front propagation velocity.
v_0 = specific volume of medium ahead of shock front.
v = specific volume of medium behind shock front.
$\bar{\beta}_0$ = mean compressibility at zero pressure over temperature range ΔT.
ρ = density.
σ = Riemann function: $\int_{p_0}^{p} \frac{v[p', S]}{c[p', S]}dp'$.
ω = undissipated enthalpy increment: $\int_{p_0}^{p} v[p', S]dp'$.

APPENDIX B

COMPUTER LISTING

```
01 LBL ^THH
02 ^TLOOP?
03 PROMPT
04 STO 34
05 ^TN?
06 PROMPT
07 STO 28
08 1
09 +
10 RCL 28
11 1
12 -
13 /
14 STO 21
15 1
16 RCL 28
17 1/X
18 -
19 STO 22
20 RCL 28
21 1/X
22 CHS
23 STO 23
24 ^TTEMP 0?
25 PROMPT
26 STO 05
27 ^TPRESSURE?
28 PROMPT
29 STO 12
30 RCL 05
31 XEQ ^TVW
32 STO 27
33 LBL 01
34 XEQ ^TY
35 STO 25
36 RCL 01
37 XEQ ^TVW
38 STO 26
39 RCL 01
40 XEQ ^TSPEED
41 STO 20
42 RCL 01
43 XEQ ^THT
44 STO 06
45 RCL 25
46 RCL 22
47 Y↑X
48 1
49 -
50 RCL 21
51 X
52 RCL 25
53 -
54 CHS
55 RCL 25
56 RCL 23
57 Y↑X
58 -
59 2
60 /
61 RCL 20
62 X
63 RCL 26
64 RCL 27
65 -
66 2
67 /
68 RCL 20
69 X
70 RCL 26
71 /
72 RCL 25
73 1
74 -
75 X
76 -
77 STO 07
78 ^THH =
79 ARCL X
80 AVIEW
81 STOP
82 RCL 06
83 1000
84 X
85 ^THT =
86 ARCL X
87 AVIEW
88 STOP
89 RCL 07
90 RCL 06
91 1000
92 X
93 -
94 ^THH-HT =
95 ARCL X
96 AVIEW
97 STOP
98 ABS
99 RCL 34
100 X<>Y
101 X<Y?
102 GTO ^TSHOCK
103 GTO 01
104 LBL ^TVW
105 25
106 -
107 STO 03
108 ENTER↑
109 X
110 3.241E-6
111 X
112 RCL 03
113 2.929E-4
114 X
115 +
116 0.99415
117 +
118 RTN
119 LBL ^TY
120 ^TTEMP 1?
121 PROMPT
122 STO 01
123 XEQ ^TBT
124 RCL 08
125 1/X
```

```
126 RCL 12
127 X
128 1
129 +
130 STO 25
131 RTN
132 LBL^T SPEED
133 STO 01
134 XEQ^T BT
135 STO 11
136 RCL 01
137 XEQ^T VW
138 0.001
139 X
140 RCL 11
141 X
142 STO 20
143 RTN
144 LBL^T HT
145 STO 01
146 RCL 05
147 -
148 3.9644
149 X
150 RCL 01
151 ENTER↑
152 X
153 RCL 05
154 ENTER↑
155 X
156 -
157 0.000312
158 X
159 +
160 RTN
161 LBL^T BTPRIME
162 ENTER↑
163 55
164 -
165 STO 10
166 -2.362E-4
167 X
168 -0.00165
169 +
170 RCL 10
171 ENTER↑
172 X
173 1.596E-6
174 X
175 +
176 1.013E8
177 X
178 STO 24
179 RTN
180 LBL^T BETA
181 25
182 -
183 6.482E-6
184 X
185 2.929E-4
186 +
187 0.001
188 X
189 STO 19
190 RTN
191 LBL^T BT
192 55
193 -
194 ENTER↑
195 ENTER↑
196 STO 04
197 X
198 STO 02
199 X
200 5.32E-7
201 X
202 RCL 02
203 -1.181E-4
204 X
205 +
206 RCL 04
207 -0.00165
208 X
209 +
210 3.134
211 +
212 STO 08
213 1.013E8
214 X
215 STO 09
216 RTN
217 LBL^T SHOCK
218 RCL 11
219 1.013E8
220 /
221 1/X
222 RCL 12
223 X
224 1
225 +
```

```
226 RCL 28
227 1/X
228 Y↑X
229 1/X
230 RCL 26
231 X
232 STO 13
233 FIX 6
234 TV =
235 ARCL X
236 AVIEW
237 STOP
238 FIX 0
239 RCL 12
240 1.013E8
241 X
242 STO 15
243 RCL 27
244 RCL 13
245 -
246 0.001
247 X
248 X
249 SQRT
250 STO 14
251 TWATER U =
252 ARCL X
253 AVIEW
254 STOP
255 RCL 15
256 RCL 27
257 0.001
258 X
259 X
260 RCL 14
261 /
262 STO 16
263 TSHOCK U =
264 ARCL X
265 AVIEW
266 STOP
267 RCL 20
268 RCL 28
269 X
270 SQRT
271 STO 17
272 RCL 26
273 RCL 13
274 /
275 RCL 28
276 1
277 -
278 2
279 /
280 Y↑X
281 X
282 STO 18
283 TWAVE C =
284 ARCL X
285 AVIEW
286 STOP
287 GTO TDEL T
288 RTN
289 LBL TCP
290 0.624
291 X
292 3964.4
293 +
294 STO 29
295 RTN
296 LBL TG1
297 RCL 05
298 273.16
299 +
300 RCL 24
301 X
302 RCL 27
303 0.001
304 X
305 X
306 RCL 28
307 1
308 -
309 /
310 RCL 29
311 /
312 STO 30
313 RTN
314 LBL TD2
315 RCL 28
316 RCL 08
317 X
318 RCL 19
319 X
320 RCL 24
321 1.013E8
322 /
323 /
324 RCL 27
325 0.001
```

```
326 X
327 /
328 STO 31
329 RTN
330 LBL^TDEL T
331 RCL 12
332 RCL 08
333 /
334 1
335 +
336 STO 32
337 RCL 05
338 XEQ^TCP
339 RCL 05
340 XEQ^TBTPRIME
341 RCL 05
342 XEQ^TBETA
343 XEQ^TG1
344 XEQ^TD2
345 RCL 32
346 RCL 28
347 1/X
348 Y↑X
349 STO 33
350 RCL 28
351 RCL 31
352 +
353 X
354 CHS
355 RCL 28
356 +
357 1
358 -
359 RCL 32
360 RCL 31
361 1
362 +
363 X
364 +
365 RCL 30
366 X
367 RCL 33
368 /
369 FIX 2
370 ^TDEL T =
371 ARCL X
372 AVIEW
373 RTN
374 END
```

APPENDIX C

CALCULATION OF TEMPERATURE, INTERNAL ENERGY, AND ENTHALPY FOR ISENTROPIC FLOW

In order to calculate the flow field downstream of the shock wave, as illustrated in Figure C-1, one needs to calculate the internal energy and enthalpy of the water as it expands. The energy equation appropriate for the flow illustrated in Figure C-1 is

$$\frac{\partial}{\partial x}[\rho u(e + \frac{u^2 + v^2}{2}) + pu] + \frac{\partial}{\partial r}[\rho v(e + \frac{u^2 + v^2}{2}) + pv] + \frac{1}{r}[\rho v(e + \frac{u^2 + v^2}{2}) + pv] = 0 \qquad \text{(C-1)}$$

Equation (C-1), which assumes inviscid flow without heat conduction, indicates the need to calculate internal energy.

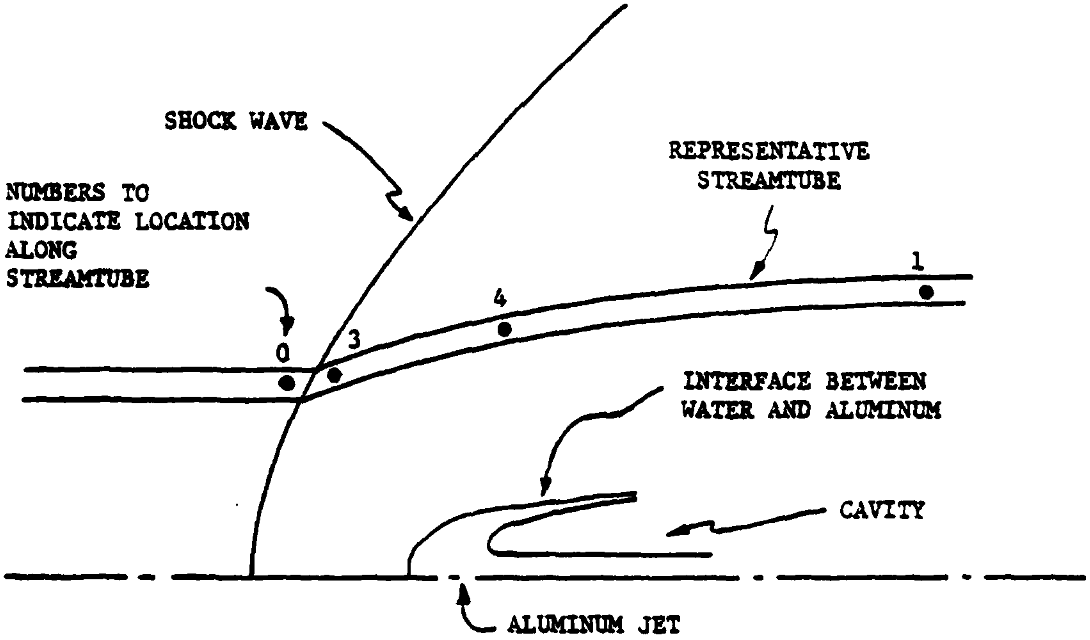


Figure C-1. Sketch of Flow Field Due to an Aluminum Jet Penetrating Water.

A. Calculation of Internal Energy and Enthalpy

By definition internal energy and enthalpy are related by

$$H = e + pv \qquad \text{(C-2)}$$

Capital H is used for enthalpy since h is used for dissipated enthalpy. Refer to Figure C-2 which is similar to Figure 1 of Appendix A. The points indicated in Figure C-1 are shown also in Figure C-2. For example, in Figure C-1 point 3 is immediately downstream of the shock wave. In passing, one should note that to calculate shock properties, the normal component of velocity should be used. Point 3 is shown in Figure C-2. As water flows along the streamtube, expansion occurs. The pressure decreases as indicated by point 4 in Figure C-1 or Figure C-2.

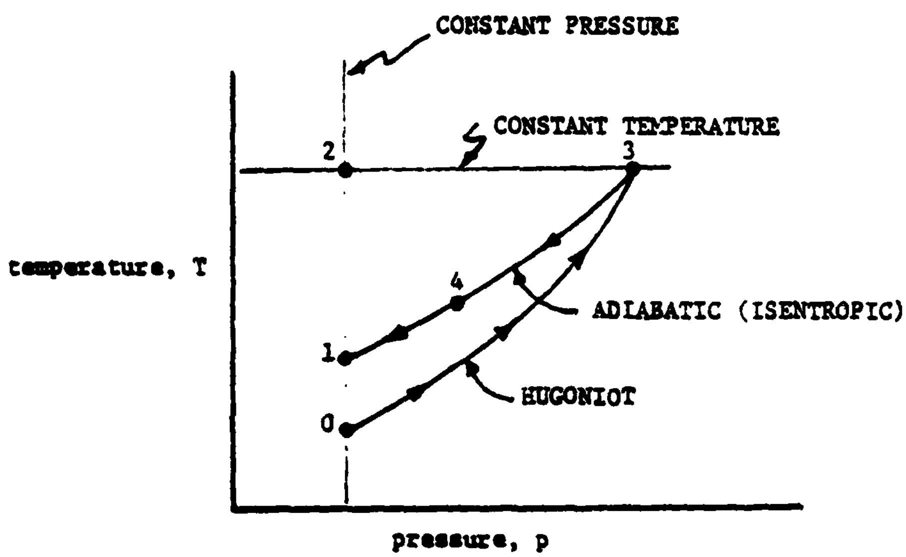


Figure C-2. Hugoniot and Adiabatic (Isentropic) Curves in the Pressure-Temperature Plane.

From equation 2.20 of Appendix A

$$\Delta H = H_3 - H_0 = \omega_{31} + h_{10} \tag{C-3}$$

The subscripts 31, used above and later in other equations, mean ω is evaluated along the adiabatic curve between points 3 and 1. Likewise h_{10} is evaluated along a line of constant pressure between points 1 and 0 in Figure C-2. From equation (2.19) of Appendix A

$$\omega_{31} = \frac{c_1^2}{n - 1}\left[\left(\frac{v_1}{v_3}\right)^{n-1} - 1\right] \tag{C-4}$$

Also, from equation (2.20) of Appendix A,

$$h_{10} = \int_0^1 C_p(0,T')dT' \tag{C-5}$$

At some intermediate point, such as point 4 in Figure C-2, one replaces subscript 3 by 4 in equations (C-4) and (C-5).

Combining equations (C-2) through (C-5), an expression for internal energy e can be derived; the result is

$$e = h_{10} + c_1^2\left[\frac{z^{n-1}}{n(n - 1)} + \frac{1}{nz} - \frac{1}{n - 1}\right] \tag{C-6}$$

where z is v_1/v. Hence equation (C-6) uses v as the independent variable. An alternate form can be derived using p as the independent variable; the result is

$$e = h_{10} + c_1^2\left[\frac{(1 + x)^{\frac{n-1}{n}}}{n(n - 1)} + \frac{1}{n(1 + x)^{1/n}} - \frac{1}{n - 1}\right] \tag{C-7}$$

where $x = p/B = p/A$. The symbols B and A are defined on page 22, Appendix A; see equation (II-2). Equations (C-6) and (C-7) have a very similar form. An universal equation can be written as

$$e = h_{10} + c_1^2\left[\frac{w^{n-1}}{n(n - 1)} + \frac{1}{nw} - \frac{1}{n - 1}\right] \tag{C-8}$$

where

$$w = \begin{cases} z \text{ for } v_1/v \text{ as independent variable} \\ (1 + x)^{1/n} \text{ for } p/B \text{ as independent variable} \end{cases} \qquad \text{(C-9)}$$

Note that h_{10} is a constant value for any streamtube.

B. Calculation of Temperature

If the energy equation incorporates thermal conduction, a term involving temperature is used. The term has the form

$$Q = k\frac{\partial T}{\partial x} \qquad \text{(C-10)}$$

where Q is the heat flux in Joule/m^2 sec, and k is thermal conductivity.

To calculate the temperature at some intermediate point, i.e. T_4, one can use the subroutine DEL T. In DEL T, p_3 is replaced by p_4. As a note of caution, when intermediate values of temperature are calculated, the pressure behind the shock wave, p_3, is replaced in storage register 12 by p_4, the intermediate pressure.

C. Subroutine E-BAR and User's Guide to E-BAR

A subroutine has been written to calculate internal energy and enthalpy. A listing of the subroutine is given below:

```
01 LBL^TE-BAR     11 Y↑X       21 -
02 ^TISEN P?      12 STO 35    22 /
03 PROMPT         13 RCL 28    23 RCL 28
04 STO 36         14 1         24 1/X
05 RCL 08         15 -         25 RCL 35
06 /              16 Y↑X       26 /
07 1              17 RCL 28    27 +
08 +              18 /         28 RCL 28
09 RCL 28         19 RCL 28    29 1
10 1/X            20 1         30 -
```

```
31 1/X
32 -
33 RCL 17
34 ENTER↑
35 X
36 X
37 RCL 07
38 +
39 TE-BAR =
40 ARCL X
41 AVIEW
42 STO 37
43 STOP
44 RCL 36
45 1.013E8
46 X
47 RCL 26
48 0.001
49 X
50 X
51 RCL 35
52 /
53 RCL 37
54 +
55 TH BAR =
56 ARCL X
57 AVIEW
58 STO 38
59 END
```

As a convenience, subroutine E-BAR should be assigned to some key when calculator is in USER mode.

To operate E-BAR, one must execute HH as discussed on pages 4 to 9. Operation of HH fills the registers with values which are recalled for use by E-BAR. For a sample problem, HH was executed using n = 7.15, p = 90 kilobars, and t_0 = 0. When HH has been completed, one proceeds as follows:

1. Continue with HP41CV in USER mode.
2. Assign E-BAR to a key on the keyboard by pressing

   $\boxed{}$ ALPHA E-BAR ALPHA, R↓

   In this example, E-BAR has been assigned to the R↓ key.
3. Press R↓, and observe ISEN P?. This is the isentropic pressure at point 4 in Figure C-2; obviously one can select any value. The number in parentheses is the value used in the sample problem (60 kilobars).
4. Press R/S.

5. Calculator displays E-BAR =. For the sample problem (E-BAR = 1,249,766 Joule/kg).

6. Press R/S.

7. Calculator displays H BAR =. For the example problem (H BAR = 5,449,312 Joule/kg).

Subroutine E-BAR has now been completed. If calculations are desired at other values for p_4, continue as follows:

8. Press R/S, and observe ISEN P?.

9. Input ISEN P.

10. Press R/S.

11. Etc.

D. Subroutine ISEN T and User's Guide to ISEN T

A subroutine ISEN T has been written to calculate temperature along the adiabatic (isentropic) curve of Figure C-2. A listing of the subroutine is given below:

```
01 LBL TISEN T      06 STO 39       11 TISEN T =
02 TISEN P?         07 RCL 01       12 ARCL X
03 PROMPT           08 +            13 AVIEW
04 STO 12           09 273.16       14 END
05 XEQ TDEL T       10 +
```

To operate ISEN T, one must execute HH as discussed on pages 4-9. For a sample problem, HH has been executed with n = 7.15, p = 90 kilobars, and $t_0 = 0$.

1. Continue with HP41CV in USER mode.

2. Assign ISEN T to a key on the keyboard by pressing

   [ ] ALPHA ISEN T ALPHA, EEX

   In this example, ISEN T has been assigned to EEX key.

3. Press EEX, and observe ISEN P?. This is the isentropic pressure at point 4 in Figure C-2. Since p_4 or ISEN P is an independent variable, any value can be selected. The number in parentheses is the number used in the sample problem (60 kilobars).

4. Press R/S.

5. Calculator displays ISEN T =. For the sample problem, the result is (ISEN T = 740.6). The dimensions are ^{o}K.

Subroutine ISEN T has now been completed. If calculations are desired at other values for p_4, continue as follows:

6. Press R/S, and observe ISEN P?.

7. Input ISEN P (50 kilobars).

8. Press R/S.

9. Observe ISEN T = 691.7^{o}K.

10. Etc.

E. Additional Assignment of Storage Registers for Subroutines E-BAR and ISEN T

Table C-I gives the additional assignment of storage registers.

Table C-I. Additional Storage Registers for E-BAR and ISEN T

Register	Symbol	Definition	Units	Programs
35	$(1 + p/B)^{1/n}$	Also symbol w in equation (C-8). Equals v_1/v.	-	E-BAR
36	p	Intermediate pressure on isentropic line.	kilobars	E-BAR
37	e	Internal energy.	Joule/kg	E-BAR
38	H	Enthalpy.	Joule/kg	E-BAR
39	ΔT (Isentropic)	Change in temperature along the adiabatic line in Figure C-2.	^{o}K	ISEN T

F. Sample Results for Properties Along Adiabatic (Isentropic) Line

Table C-II summarizes the results for conditions along the adiabatic line in Figure C-II. Input values for the shock wave were n = 7.15, p = 90 kilobars, $t_0 = 0^{o}C$, and $t_1 = 167.762^{o}C$.

Table C-II. Sample Results for Conditions Along Adiabatic (Isentropic) Line

p kilobars	e kilojoule/kg	H kilojoule/kg	T ^{o}K
0	674	674	441
5	707	1180	460
10	755	1634	486
20	857	2474	539
30	958	3260	591
40	1057	4013	642
50	1154	4741	692
60	1249	5449	741
70	1343	6142	789
80	1435	6820	836
90	1526	7488	883

For p = 0 kilobars, the temperature T is the residual due to entropy increase across the shock wave. For p = 90 kilobars, the temperature T is the value immediately behind the shock wave.

REFERENCES

1. G. K. Hartmann and E. G. Hill, Editors, Underwater Explosion Research, A Compendium of British and American Reports, Volume I, "The Shock Wave," Office of Naval Research, 1950, 1585 pp.

2. J. M. Richardson, A. B. Arons, and R. R. Halverson, "Hydrodynamic Properties of Sea Water at the Front of a Shock Wave," Journal of Chemical Physics, Volume 15, pp. 758-794. 1947.

3. R. H. Cole, Underwater Explosions, Princeton University Press, 1948.

4. M. Holt, "Underwater Explosions," Annual Review of Fluid Mechanics, Volume 9, Annual Reviews, Inc., Palo Alto, CA, pp. 187-214, 1977.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
2.	Library, Code 0142 Naval Postgraduate School Monterey, CA 93940	2
3.	Chairman Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93940	1
4.	Chairman Department of Physics and Chemistry Naval Postgraduate School Monterey, CA 93940	1
5.	Chairman Department of Aeronautics Naval Postgraduate School Monterey, CA 93940	1
6.	Mr. Donald Phillips Naval Surface Weapons Center Silver Spring, MD 20910	2
7.	Mr. Nat Coleburn Naval Surface Weapons Center Silver Spring, MD 20910	1
8.	Mr. Wayne Reed Naval Surface Weapons Center Silver Spring, MD 20910	1
9.	Mr. George Daniello Naval Surface Weapons Center Silver Spring, MD 20910	1
10.	LT Jerry Buck, USN c/o Mr. Nat Coleburn Naval Surface Weapons Center Silver Spring, MD 20910	1
11.	LT John Strott, USN c/o Mr. Nat Coleburn Naval Surface Weapons Center Silver Spring, MD 20910	1

12. Professor Richard A. Reinhardt 1
Code 61Ri
Naval Postgraduate School
Monterey, CA 93940

13. Professor Allen E. Fuhs 50
Code 67Fu
Department of Aeronautics
Naval Postgraduate School
Monterey, CA 93940

14. Dr. Paul Kutler 1
NASA Ames Research Center, MS 202A-14
Moffett Field, CA 94035

15. Professor Maurice Holt 1
Department of Mechanical Engineering
University of California
Berkeley, CA 94720

16. LCDR Amos Hallenbeck, USN 1
c/o Code 31
Naval Postgraduate School
Monterey, CA 93940

17. Dr. Frank Hains 1
Naval Surface Weapons Center
Silver Spring, MD 20910

DATE
FILMED

4-82

DTIC

ONT

AD-A112 040 NAVAL POSTGRADUATE SCHOOL MONTEREY CA F/G 20/4
COMPUTER PROGRAM FOR CALCULATING HYDRODYNAMIC PROPERTIES OF SHO--ETC(U)
FEB 82 A E FUHS
UNCLASSIFIED NPS67-82-001 NL

2 OF 2
AD A
112040

END
DATE
FILMED
07-82
DTIC

A

204

SUPPLEMENTARY

INFORMATION

NAVAL POSTGRADUATE SCHOOL

MONTEREY, CALIFORNIA - 93940

IN REPLY REFER TO

Code 67
AEF:ef

AD-A112040

6 April 1982

From: Professor Allen E. Fuhs

To: Distribution List

Subj: Supplement to NPS Technical Report 67-82-001

Ref: (a) NPS Report 67-82-001, February 1982

As the needs for computational flexibility have changed, the report of Reference (a) has grown. Enclosed is Appendix D, which permits calculation of pressure as a function of specific volume and internal energy. An iteration is required, and an efficient iteration procedure has been developed. A fractional error for the iteration variable of 5E-05 can be achieved in 2 or 3 iterations.

Please add Appendix D to your copy of Reference (a).

APPENDIX D
(NPS67-82-001)

CALCULATION OF PRESSURE AS A FUNCTION OF SPECIFIC VOLUME AND INTERNAL ENERGY

Introduction

In order to use efficiently the computer program developed by Kutler and Chakravarthy [D-1], the ability to calculate pressure, p, as a function of specific volume, v, and internal energy, $\bar{e}$, is needed. The program for HP41CV, which is described in this appendix, calculates $p = p(v,\bar{e})$.

Relevant Equations

The relevant equations for the calculation can be assembled from the equations in this report. Define

$$z = v_1/v \tag{D-1}$$

where v_1 is the specific volume at t_1, and v is the specific volume at the desired condition. The value for v_1 can be calculated from

$$v(t) = 0.99415 + 0.0002929(t - 25) + 3.241 \times 10^{-6}(t - 25)^2 \tag{D-2}$$

The internal energy is given by

$$\bar{e} = h_{10} + c_1^2\left[\frac{z^{n-1}}{n(n - 1)} + \frac{1}{nz} - \frac{1}{n - 1}\right] = \bar{e}(t_1,v_1/v) \tag{D-3}$$

Note that $\bar{e}$ is a function of t_1 and v_1/v. The quantity h_{10} is

$$h_{10} = \int_{t_0}^{t_1} C_p(0,t)dt = 3.9644(t_1 - t_0) + 3.12 \times 10^{-4}(t_1^2 - t_0^2) \tag{D-4}$$

The temperature, t_0, is the reference temperature which can be the initial water temperature before the shock wave arrives.

The pressure can be related to specific volume by the Tait equation

$$p = B(t_1)[(v_1/v)^n - 1] = p(v,t_1) \tag{D-5}$$

The parameter B(t) is given by

$$B(t) = 3.134 - 1.65 \times 10^{-3}(t - 55) - 1.181 \times 10^{-4}(t - 55)^2 + 5.32 \times 10^{-7}(t - 55)^3 \qquad \text{(D-6)}$$

Finally for equation (D-3), the value of c_1^2 must be calculated. The appropriate formula is

$$c_1^2 = nv_1B(t_1) \qquad \text{(D-7)}$$

The value of n is 7.15.

Iteration Procedure

The preceding equations permit calculation of $p = p(\overline{e},v)$; however, an explicit formulation is not possible, or, at least, is very difficult. Consequently, an iterative approach is used. Define $\overline{e}_c$ as a calculated value from equation (D-3). Define $\overline{e}_i$ as an input value at which the pressure is desired. As implied by Figure D-1, an iterative procedure is needed to find the correct value for t, at which $\overline{e}_c = \overline{e}_i$.

The function $\overline{e}_c$ is a monotonically increasing function in the temperature region of interest. An iteration scheme using straight lines is used to find t_1. For computer programming, capital letters are used; hence, in terms of capital letters, the equation for the straight line is

$$T3,J = \frac{\overline{E} - E1}{E2 - E1}(T2 - T1) + T1 \qquad \text{(D-8)}$$

where the symbol definitions are as follows:

T3,J calculated value for temperature using a straight line as shown in Figure D-2; J is an index which indicates the number of the iteration.

$\overline{E}$ desired value of E BAR; also identified as input value $\overline{e}_i$.

E1 value of E BAR at temperature T1.

E2 value of E BAR at temperature T2.

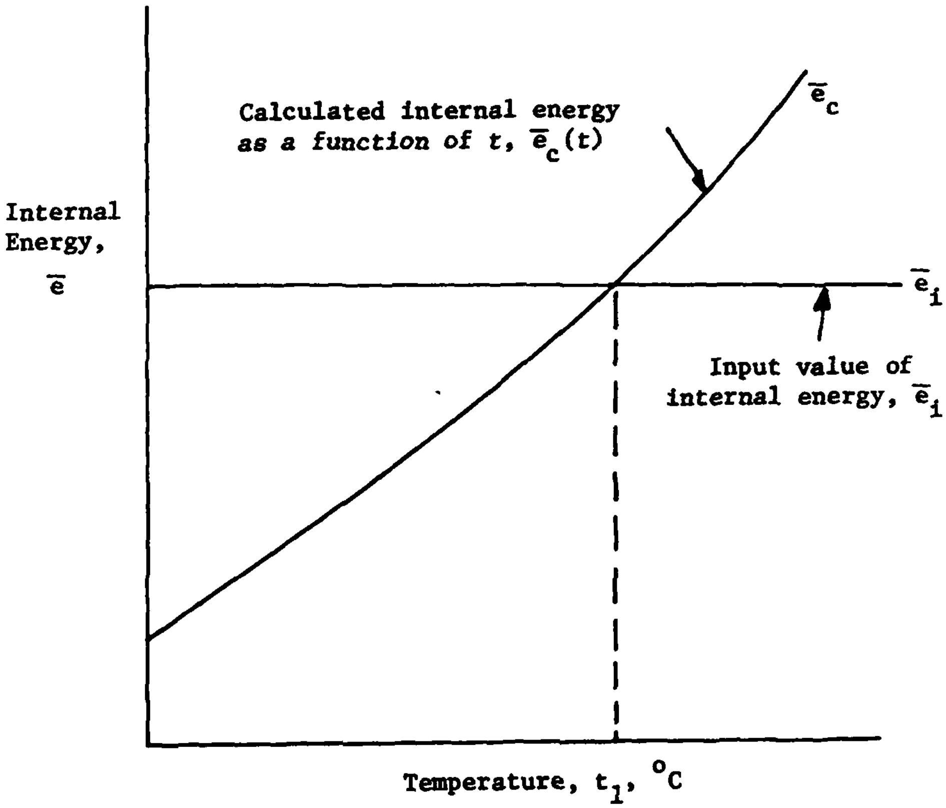
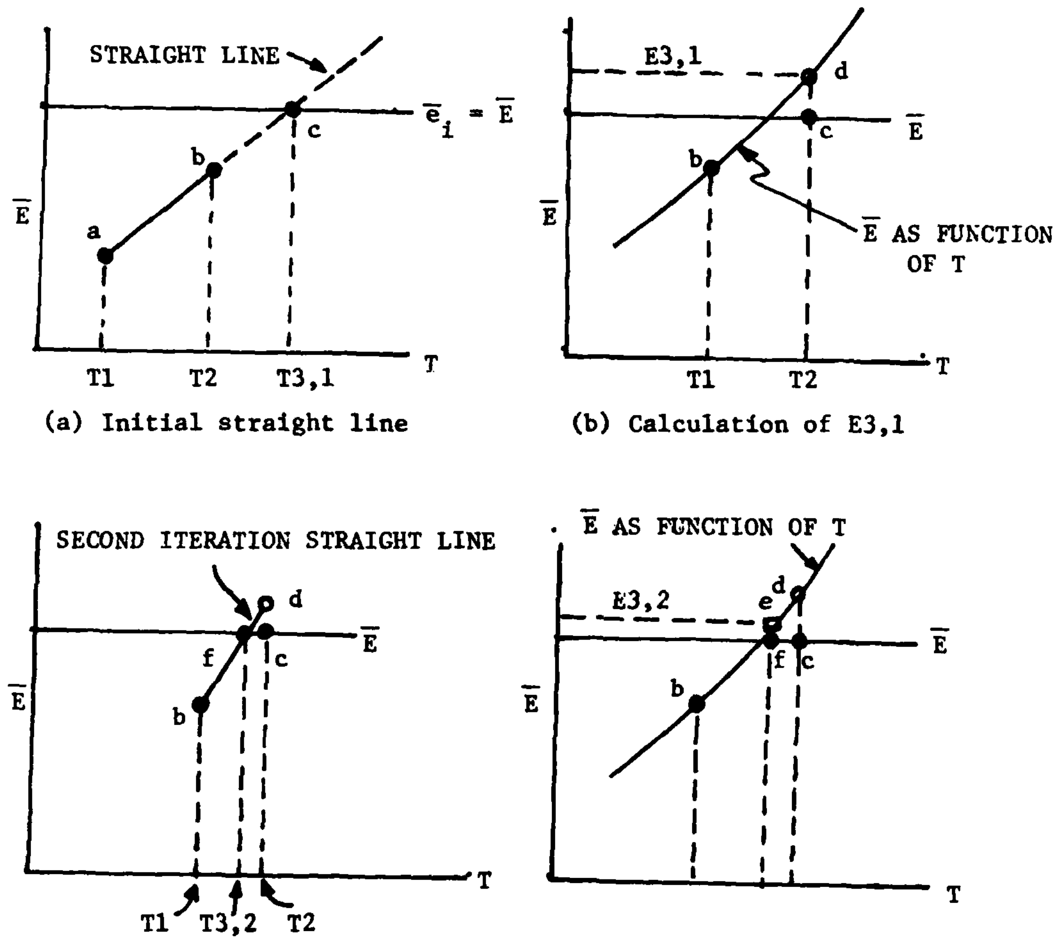


Figure D-1. Iteration to determine correct value of t_1.



(a) Initial straight line

(b) Calculation of E3,1

(c) Revised straight line and solution for T3,2

(d) Calculation of E3,2

Figure D-2. Illustration of iteration procedure.

Refer to Figure D-2(a). The first values for T1 and T2 are inputs and are guesses. The program calculates E1 and E2 and solves for T3,1 as shown in Figure D-2(a). The value for T3,1 is used to calculate E3. E3 is compared to $\overline{E}$. If the difference, $\overline{E}$ - E3, is too large, then T2 = T3,1, T1 = T2, E2 = E3, and E1 = E2. A new straight line is calculated passing through points b and d in Figure D-2(b) and Figure D-2(c). A solution is obtained for T3,2, which is the second value for T3. Using T3,2, E3 is calculated and is once again compared with $\overline{E}$. If the quantity ABS($\overline{E}$ - E3) is too large, the values for E and T are reassigned as follows: T2 = T3,2, T1 = T2, E2 = E3, and E1 = E2.

Once again a solution is obtained for T3,3 using a line passing through points d and e in Figure D-2(d). The procedure repeats until the desired accuracy for ABS($\overline{E}$ - E3) is obtained.

Program Description

Figure D-3 is a flow chart for the program $p = p(\overline{e},v)$, which has been labelled P EV. Additional storage registers are assigned as shown in Table D-1. The assignment of registers 1 to 39 is given in Table 1 and Table C-1. Table D-2 is a program listing. The various subroutines, such as VW, BT, and H10, must be in the memory of HP41CV.

As shown in the flow chart of Figure D-3, inputs are made for B, E BAR, J, T1, and T2. An input J = 0 must always be used; otherwise the program logic will be in disarray. The quantities T and V are the variables used in subroutine LBL 12 to calculate E. The quantities utilized or calculated as a function of J are as follows:

VALUE OF J	VALUE OF T	VALUE OF E
0	T1	E1
1	T2	E2
2	T3	E3

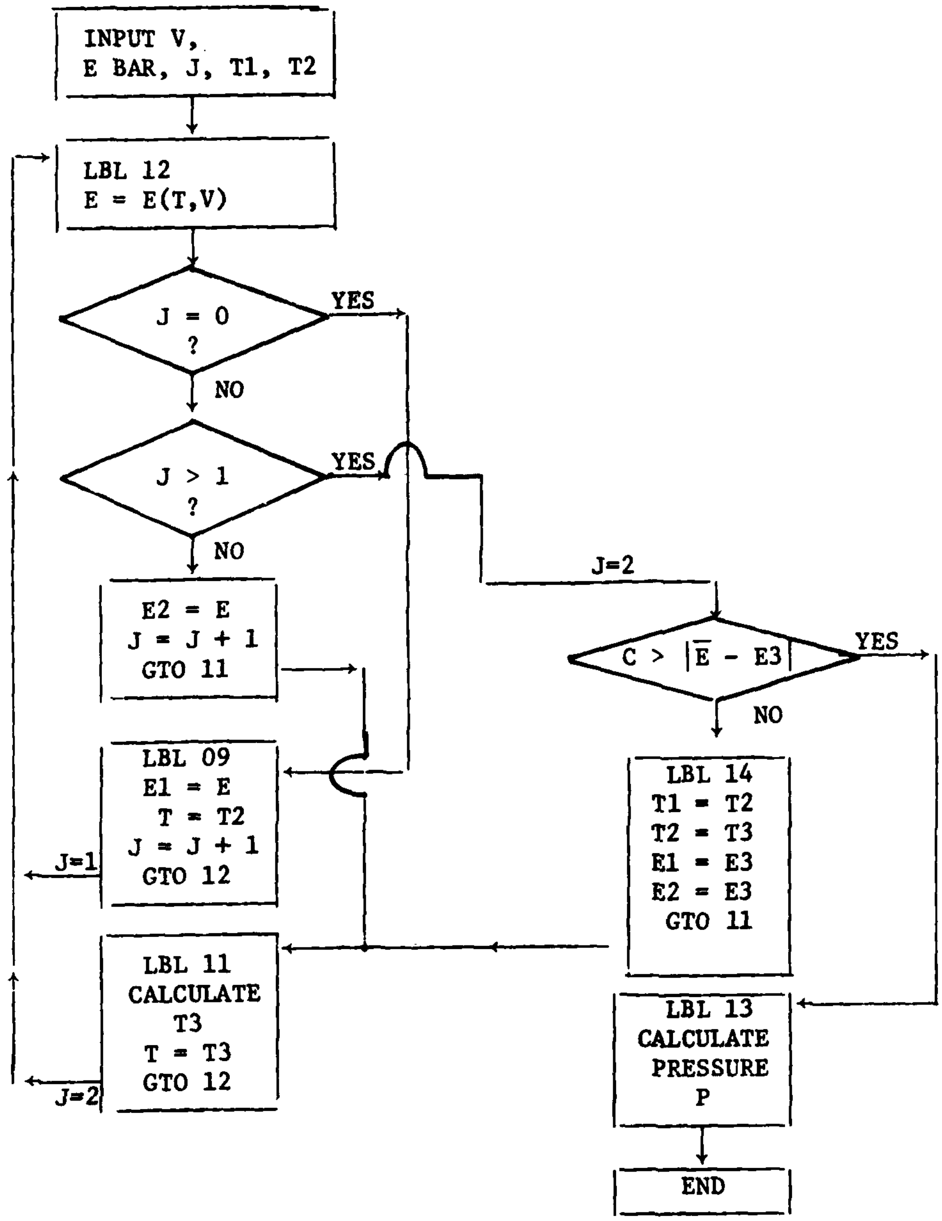


Figure D-3. Flow chart for the program P EV.

Table D-1. Additional Storage Registers for P EV

Register	Symbol	Definition	Units	Programs
40	v	Specific volume at which pressure is desired	cm^3/gm	12
41	$\bar{E}$	Internal energy at which pressure is desired	kilojoule/kg	12
42	T	Value of t, as used in equation (C-5)	^oC	VW, BT
43	-	Not assigned	-	-
44	v_1	Value of specific volume at $p = 0$ and temperature t_1	cm^3/gm	12, 13
45	$B(t)$	Parameter in equation of state relating p to v	kilobars	12, 13
46	c_1^2	Speed of sound squared at temperature t_1	m^2/sec^2	12
47	h_{10}	Contribution to enthalpy at zero pressure between temperatures t_0 and t_1	kilojoule/kg	12
48	z	Ratio v_1/v	-	12
49	E1	Internal energy evaluated at T1	Joule/kg	12
50	E2	Internal energy evaluated at T2	Joule/kg	12
51	T1	Temperature at point 1 on straight line	^oC	12
52	T2	Temperature at point 2 on straight line	^oC	12
53	J	Index to count number of executions of LBL 12	-	09
54	E	Register to store output of LBL 12 which is $E = E(T,v)$	Joule/kg	12
55	T3	Temperature of point determined by straight line	^oC	11
56	E3	Internal energy at temperature T3	Joule/kg	12, 14

Table D-2. Program Listing

01 LBL^{T}P EV	26 XEQ^{T}BT	51 1/X	76 RCL 53	101 RCL 51	126 RCL 50
02 ^{T}V?	27 STO 45	52 7.15	77 1	102 -	127 STO 49
03 PROMPT	28 7.15	53 /	78 +	103 X	128 RCL 56
04 STO 40	29 X	54 +	79 STO 53	104 RCL 51	129 STO 50
05 ^{T}E BAR?	30 RCL 44	55 6.15	80 GTO 11	105 +	130 GTO 11
06 PROMPT	31 X	56 1/X	81 LBL 09	106 ^{T}T3 =	131 LBL 13
07 1000	32 1000	57 -	82 RCL 54	107 ARCL X	132 7.15
08 X	33 /	58 RCL 46	83 STO 49	108 AVIEW	133 RCL 48
09 STO 41	34 STO 46	59 X	84 RCL 52	109 STOP	134 X<>Y
10 ^{T}J?	35 RCL 42	60 RCL 47	85 STO 42	110 STO 55	135 Y↑X
11 PROMPT	36 XEQ^{T}H 10	61 +	86 RCL 53	111 STO 42	136 1
12 STO 53	37 1000	62 ^{T}E =	87 1	112 GTO 12	137 -
13 ^{T}T1?	38 X	63 ARCL X	88 +	113 LBL 14	138 RCL 45
14 PROMPT	39 STO 47	64 AVIEW	89 STO 53	114 RCL 54	139 X
15 STO 51	40 RCL 44	65 STOP	90 GTO 12	115 STO 56	140 1.013E8
16 ^{T}T2?	41 RCL 40	66 STO 54	91 LBL 11	116 RCL 41	141 /
17 PROMPT	42 /	67 RCL 53	92 RCL 50	117 -	142 ^{T}PRESS =
18 STO 52	43 STO 48	68 X=0?	93 RCL 49	118 ABS	143 ARCL X
19 RCL 51	44 6.15	69 GTO 09	94 -	119 50	144 AVIEW
20 STO 42	45 Y↑X	70 1	95 RCL 41	120 X>Y?	145 END
21 LBL 12	46 7.15	71 RCL 53	96 RCL 49	121 GTO 13	
22 RCL 42	47 /	72 X>Y?	97 -	122 RCL 52	
23 XEQ^{T}VW	48 6.15	73 GTO 14	98 X<>Y	123 STO 51	
24 STO 44	49 /	74 RCL 54	99 /	124 RCL 55	
25 RCL 42	50 RCL 48	75 STO 50	100 RCL 52	125 STO 52	

Once J has acquired a value of 2, the program loops repeatedly as follows: LBL 12, TEST ABS($\bar{E}$-E3), LBL 14, LBL 11, LBL 12, etc. When the criterion for the value of ABS($\bar{E}$-E3) is attained, the program departs from the loop and moves to LBL 13.

Sample Run

The program P EV has been assigned a key in USER mode of HP41CV. Pushing that key causes the following queries to appear:

V?	0.75 cm^3/gm
E BAR?	900 kilojoule/kg
J?	0
T1?	50^oC
T2?	150^oC

Once the input has been made, the crow's foot moves across the display indicating the program is being executed. Next to appear, after R/S is pushed, is

E = 377,507 (Joule/kg)

which is the value for E1 calculated at T1. Next

E = 880,354 (Joule/kg)

appears which is E2. The situation for the straight line is exactly as shown in Figure D-2(a). Pushing R/S yields

T3 = 153.91 (= T3,1^oC)

Pushing R/S causes the program to display

E = 902,126

which is E3. The situation is that illustrated in Figure D-2(b). The sequence of values is listed below

T3 = 153.53 (= T3,2^oC)

E = 899,985

PRESS = 30.45 (kilobars)

Reference to line 119 of Program Listing indicates that

$$|\bar{E}-E3|<50$$

has been achieved.

To indicate the influence of the initial values for T1 and T2, the program was run again with

$$V = 0.75\ cm^3/gm$$

$$E\ BAR = 900\ kilojoule/kg$$

$$J = 0$$

$$T1 = 0$$

$$T2 = 10$$

The sequence of values is listed below:

$$E = 135{,}345\ (= E1)$$

$$E = 184{,}447\ (= E2)$$

$$T3 = 155.73\ (= T3,1)$$

$$E = 912{,}390\ (= E3)$$

$$T3 = 153.25\ (= T3,2)$$

$$E = 898{,}425$$

$$T3 = 153.53\ (= T3,3)$$

$$E = 899{,}993$$

$$PRESS = 30.45$$

Using values of T1 and T2 quite far removed from final value of T3,J, the program converged to a solution in three iterations.

Reference

D-1. P. Kutler and S. R. Chakravarthy, "Supersonic Flow over Ablated Nosetips Using an Unsteady Implicit Numerical Procedure," AIAA 78-213, 1978.

FILMED

7-8